International Journal of Tomography & Statistics
Summer 2007, Vol. 6, No. S07; Int. J. Tomogr. Stat.; 63-68
ISSN 0972-9976;

COMBINING $\mathcal{H}_\infty$ CONTROL AND DYNAMIC INVERSION FOR ROBUST CONSTRAINED SET-POINT REGULATION

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Abstract: A new method for the constrained robust set-point regulation of scalar linear systems is proposed in this paper. It is based on the combined synthesis of the feedback controller and of the closed-loop command function. In particular, the controller is synthesized by solving a standard $\mathcal{H}_\infty$ mixed sensitivity problem and the command function is determined by solving a stable input-output inversion problem. The optimal design parameters are found by means of a genetic algorithm, so that the worst-case settling time is minimized, subject to amplitude constraints on the control variable and on the resulting undershoot and overshoot. Worked examples are given to illustrate the methodology.

Keywords: Robust set-point regulation, $\mathcal{H}_\infty$ control, inversion, optimisation, constraints.

1. INTRODUCTION

It is well-known that the regulation performance provided by a feedback control system can be improved by adopting an inversion-based feedforward controller when the model uncertainties are sufficiently small (Devasia, 2002). In this context, the typical design approach is to determine a feedforward controller based on the nominal model of the plant and then to design independently a feedback controller in order to cope with modelling errors and initial condition mismatches, in addition to external disturbances. When preview information is available, a noncausal feedforward controller can be employed, i.e. a command input is synthesized by means of appropriate stable inversion procedures (see for example (Hunt *et al.*, 1996)). This approach has been shown to be effective in practical cases (see for example (Zou and Devasia, 2004)), though, in general, the determined command input exhibits a pre- and a post-actuation time intervals.

From a different point of view, a design approach based on a combined synthesis of the feedback controller and of the noncausal feedforward controller has been proposed in (Piazzi and A.Visioli, 2001), where the command input function is determined by applying a stable input-output inversion procedure to the nominal complementary sensitivity function. The rationale of this method is that of exploiting the capability of the feedback controller of reducing the effects of the model uncertainties in a range of frequencies and therefore the degradation of the set-point regulation performance due to the model mismatch is reduced.

Following this basic idea, in this paper we formulate a more general problem (see Section 2) and we propose a solution based on the use of the $\mathcal{H}_\infty$ control theory (see for example (Zhou, 1998)). Specifically, a set of plants is considered and the feedback controller is designed by solving a standard $\mathcal{H}_\infty$ mixed-sensitivity problem (Safonov *et al.*, 1989), where a parameterized weighting function is chosen for the sensitivity function. Then, a transition polynomial parameterized by the transition time (Piazzi and Visioli, 2001*a*) is chosen as a desired output function and a stable input-output inversion procedure (Piazzi and Visioli, 2005) is solved with respect to the nominal plant. The design parameters are chosen by means of a genetic algorithm in order to minimize the worst-case settling time by taking into account bounds on the control variable and arbitrarily chosen limits on the maximum overshoot and un-

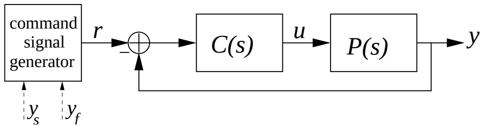

Fig. 1. The unity-feedback system.

dershoot of the output.

Roughly speaking, the method aims at minimizing the sensitivity function (i.e. minimizing the effect of the plant uncertainties) in a certain range of frequencies (while guaranteeing the robust stabilization) and then to apply a command signal whose frequency content is in that range. It is the genetic algorithm that implicitly addresses this concept by selecting the most appropriate value of the parameters.

2. PROBLEM FORMULATION

We consider a set of scalar systems

$$\mathbf{P} = \{P_1(s), P_2(s), \ldots, P_n(s)\} \tag{1}$$

In the context of a unity-feedback system (see Figure 1), we search for a feedforward/feedback control strategy in order to obtain a "robust" transition from a previous set-point value y_s to a new one y_f. Without loss of generality in the following we will assume $y_0 = 0$ and $y_f > 0$. Obviously, the first requirement to be satisfied is the stability of the closed-loop system for all plants P_i, $i = 1, \ldots, n$. Moreover, this transition has to satisfy an overshoot and an undershoot limitation, an amplitude constraint on the control variable $u(t)$ and has to minimize the (worst-case) settling time. In other words, the above problem can be stated as follows: determine a reference function $r(t)$ and a controller $C(s)$ such that for all plants $P_i(s)$, $i = 1, \ldots, n$

(1) the closed-loop system is stable;
(2) $\lim_{t\to\infty} y(t) = y_f$ (steady-state condition);
(3) the overshoot in response to $r(t)$ is bounded by a given $\bar{O}$;
(4) the undershoot in response to $r(t)$ is bounded by a given $\bar{U}$;
(5) the absolute value of the control variable $u(t)$ is bounded by a given u_{sat};
(6) it is minimized the worst-case settling time.

Searching for the true global solution of the above problem is indeed extremely difficult. Thus, in the following we search for a practicable sub-optimal but effective solution based on the concepts of $\mathcal{H}_\infty$ control and dynamic inversion.

3. DESIGN METHODOLOGY

3.1 Controller design

As a first step, a nominal model $P_0(s)$ has to be selected for the set of considered scalar systems $\mathbf{P}$. Then, a multiplicative uncertainty

$$\Delta_i(s) := \frac{P_i(s) - P_0(s)}{P_0(s)}, \quad i = 1, \ldots, n \tag{2}$$

has to be calculated for each plant and then a transfer function $W_s(s)$ has to be determined so that $(i = 1, \ldots, n)$

$$|\Delta_i(j\omega)| \leq |W_s(j\omega)|, \quad \forall\omega \in [0, +\infty), \tag{3}$$

i.e. it represents an uncertainty bound for the nominal system.

This is actually a standard, although somewhat conservative, practice in the robust control framework. It is worth noting however that, from a practical point of view, in case of a plant affected by structured (parametric) uncertainty, P_0 can be conveniently selected as that obtained by considering, for each uncertain parameter, the midpoint values of the uncertainty interval and the P_i's can be selected as those resulting by considering the vertices of the uncertainty region.

Once W_s has been defined, a parametric weighting function W_e is defined for the sensitivity function:

$$W_e(s) = K\frac{s+a}{s} \tag{4}$$

Then, the following typical $\mathcal{H}_\infty$ mixed-sensitivity problem, which addresses the performance of the closed-loop system by ensuring the robust stability at the same time, is posed (Safonov *et al.*, 1989).

$\mathcal{H}_\infty$ **mixed-sensitivity control problem**. Find an internally stabilizing controller $C(s; K, a)$ such as:

$$\left\| \begin{matrix} W_e(j\omega)S(j\omega) \\ W_s(j\omega)T(j\omega) \end{matrix} \right\|_\infty \leq 1 \tag{5}$$

where

$$S(s; K, a) = \frac{1}{1 + C(s; K, a)P_0(s)} \tag{6}$$

and

$$T(s; K, a) = \frac{C(s; K, a)P_0(s)}{1 + C(s; K, a)P_0(s)}. \tag{7}$$

The controller $C(s; K, a)$ that solves the above problem can be determined, under the conditions specified in (Safonov *et al.*, 1989), by applying the algorithm described in (Safonov *et al.*, 1989) and implemented in the Robust Control Toolbox of Matlab (Chiang and Safonov, 1998).

Remark 1. The algorithm of Safonov et al. (Safonov *et al.*, 1989) has been preferred in this context to the Glover's and Doyle's algorithm (Glover and Doyle, 1988) as it allows to handle more easily the satisfaction of the internal model principle, i.e. the requirement of a pole at $s = 0$ in the open-loop transfer function in order to satisfy the steady-state condition. Actually, the Glover's and Doyle's algorithm provides a (sub)optimal solution to the $\mathcal{H}_\infty$ control problem (i.e. the mixed-sensitivity cost function of expression (5) is minimized), but, in the context of the methodology presented in this paper, this is not strictly necessary, as the (sub)optimal solution is provided by the genetic algorithm (see subsection 3.3).

Remark 2. It has to be noted that the conditions developed in (Safonov *et al.*, 1989) for the existence of an $\mathcal{H}_\infty$ controller might require that a weighting function is defined also for the control sensitivity transfer function, i.e. the transfer function from the command signal r to the control variable u. However this does not imply a loss of generality of the proposed methodology as an arbitrarily small weight can be selected (Chiang and Safonov, 1998).

Remark 3. If each plant in the set $\mathbf{P}$ has a pole at the origin of the complex plane, there is no need of selecting a performance weighting function such as the one of expression (4). In this case it is more indicated to select the following alternative weighting function:

$$W_e(s) = K\frac{s+a}{s+\varepsilon a} \tag{8}$$

The $\mathcal{H}_\infty$ mixed-sensitivity control problem can then be solved in this case by using a bilinear transform (Chiang and Safonov, 1992). The modifications of the methodology presented in this paper in order to address this case are then straightforward.

3.2 Command input design

The closed-loop command input is determined by applying an input-output inversion procedure to the closed-loop nominal transfer function $T(s;K,a)$. In particular, the desired output function is first designed using a parameterized "transition" polynomial $y(t;\tau)$, which determines a smooth transition between 0 and y_f, to be accomplished without undershooting and overshooting in the time interval $[0,\tau]$ (Piazzi and Visioli, 2001*a*). Function $y(t;\tau)$ can be expressed over the time interval $[0,\tau]$ by

$$y(t;\tau) = y_f\frac{(2k+1)!}{k!}\cdot \sum_{i=k+1}^{2k+1}\frac{(-1)^{i-k-1}}{(i-k-1)!(2k+1-i)!i}\left(\frac{t}{\tau}\right)^i. \tag{9}$$

Outside the transition time interval $[0,\tau]$, the desired output function is simply defined as $y(t;\tau)=0$ for $t<0$ and $y(t;\tau)=y_f$ for $t>\tau$. The order k of the polynomial is selected in order to ensure that the determined command input function is continuous. Thus, denoted by ρ the relative order of the nominal system P_0, it has to be $k=\rho$ (i.e. the order of the polynomial is $2\rho+1$).

The command function $r(t;K,a,\tau)$, to be applied to the closed-loop system in order to obtain the parameterized desired output function (9), can be determined by adopting the stable input-output inversion procedure described in (Piazzi and Visioli, 2005). The salient feature of this methodology is that it determines an easily-implementable closed-form expression of the command input $r(t;K,a,\tau)$. It is worth stressing in any case that the synthesized (bounded) command input is defined over $(-\infty,+\infty)$. Hence, in order to practically use it, it is necessary to truncate it. However, this can be done with arbitrary precision, by selecting two arbitrary small parameters ε_0 and ε_1 and by subsequently determining

$$t_0 := \max\{t'\in\mathbb{R} : |r(t;K,a,\tau)|\le\varepsilon_0 \ \ \forall t\in(-\infty,t']\}$$

and

$$t_1 := \min\{t'\in\mathbb{R} : \left|r(t;K,a,\tau)-\frac{1}{T(0;K,a)}\right|\le\varepsilon_1 \ \ \forall t\in[t',\infty)\right\}.$$

Then, by defining $t_p := \min\{0,t_0\}$ and $t_f := \max\{\tau,t_1\}$, the actual command function to be applied to the closed-loop system is given by

$$\bar{r}(t;K,a,\tau) := \begin{cases} 0 & \text{for } t<t_p \\ r(t;K,a,\tau) & \text{for } t_p\le t\le t_f \\ \dfrac{1}{y_f} & \text{for } t>t_f. \end{cases}$$

Note that when $t_p<0$ and $t_f>\tau$, the applied command function exhibits an associated pre-actuation and a post-actuation time interval respectively.

3.3 Optimization problem

In the previous subsections, a parameterized controller $C(s;K,a)$ and a parameterized command input function $\bar{r}(t;K,a,\tau)$ have been determined. Define now as $y_i(t;K,a,\tau)$ the associated output of the system $P_i(s)$, i.e. the output obtained by applying the determined command input to the closed-loop system

$$T_i(s;K,a) = \frac{C(s;K,a)P_i(s)}{1+C(s;K,a)P_i(s)} \quad i=1,\dots,n. \tag{10}$$

The corresponding (2%) settling time can then be defined as

$$t_{s,i}(K,a,\tau) := |t_p| + \min\{\bar{t}\in\mathbb{R}_+ : |y_i(t;K,a,\tau)-y_f|\le 0.02y_f \ \ \forall t\ge\bar{t}\} \tag{11}$$

and therefore the worst-case settling time is simply

$$t_{s,wc}(K,a,\tau) := \max_{i=1,\dots,n} t_{s,i}(K,a,\tau). \tag{12}$$

Define $\mathcal{C}$ as the set of parameters pairs $(K,a)\in\mathbb{R}_+^2$ for which the posed $\mathcal{H}_\infty$ mixed sensitivity control problem has a solution. Thus, the design problem formulated in Section 2 can be written as the following optimization problem:

$$\min_{K,a,\tau\in\mathbb{R}_+} t_{s,wc}(K,a,\tau) \tag{13}$$

subject to:

$$(K,a)\in\mathcal{C}; \tag{14}$$

$$y_i(t;K,a,\tau)\le(1+0.01\bar{O})y_f \ \ \forall t\ge 0 \ \ i=1,\dots,n; \tag{15}$$

$$y_i(t;K,a,\tau)\ge -0.01\bar{U}y_f \ \ \forall t\ge 0 \ \ i=1,\dots,n; \tag{16}$$

$$|u_i(t;K,a,\tau)|\le u_{sat} \ \ \forall t\ge 0 \ \ i=1,\dots,n \tag{17}$$

where $u_i(t;K,a,\tau)$ is the control variable that corresponds to the system output $y_i(t;K,a,\tau)$.

Note that the settling time definition incorporates the preaction time $|t_p|$ even though during the interval $(t_p,0)$ the output signal is almost identically zero. This appears technically sound because during the interval $(t_p,0)$ the overall system is out of equilibrium.

The following theorem ensures that the posed optimization problem has a solution (see (Safonov *et al.*, 1989)).

Theorem 1. Assume that there exist $K \in \mathbb{R}_+$ and $a \in \mathbb{R}_+$ for which the $\mathcal{H}_\infty$ mixed-sensitivity control problem described in subsection 3.1 has a solution. Then, the optimization problem (13) admits a solution if

$$u_{sat} > \left(\min_{i=1,\dots,n} |P_i(0)| \right)^{-1} y_f \tag{18}$$

Proof. It is a simple extension of a proof presented in (Piazzi and Visioli, 2001*b*). □

Remark 4. It is worth stressing again that the conditions for the existence of a solution of the $\mathcal{H}_\infty$ mixed-sensitivity control problem (5) are explained in (Safonov *et al.*, 1989).

Remark 5. In case an integral control is not needed (see Remark 3), then the condition (18) simply becomes

$$u_{sat} > 0. \tag{19}$$

An approximate solution to the above optimization problem can be affectively found by means of a genetic algorithm, where the applied constraints (15)-(17) can be easily handled by appropriately penalizing, if they are violated, the cost function represented by the worst case settling time. It is worth underlying that the rationale of the overall methodology is to minimize the sensitivity function in a range of (low) frequencies as large as possible, in order to reduce the effects of the uncertainties in the closed-loop system and therefore to increment the effectiveness of the use of the dynamic inversion. Obviously, limits are imposed by the necessity of guaranteeing the robust stability of the system (this is explicitly achieved by the condition on the complementary sensitivity function in (5)). The role of the transition time is to determine the range of frequency of the command signal so that it matches that of the sensitivity function in order to satisfy the constraints of the optimization problem (note that, roughly speaking, the more the value of τ is high, the more the actual system output is similar to the desired one (9), see (Piazzi and Visioli, 2005)).

Remark 6. An important feature of devised methodology is that the overall design can be accomplished automatically by means of appropriate software. For example, the results obtained in Section 4 have been obtained with Matlab by using the Robust Control Toolbox (Chiang and Safonov, 1998), the Genetic Algorithm Optimization Toolbox (Houck *et al.*, 1995), Simulink and the toolbox described in (Piazzi *et al.*, 2004) that implements the stable input-output inversion algorithm adopted in this paper.

4. ILLUSTRATIVE EXAMPLES

4.1 Example 1

As a first example we consider the following nonminimum-phase plant affected by structured uncertainties (Piazzi and A.Visioli, 2001):

$$P(s;\mathbf{q}) = \frac{(s-q_1)(s-q_2)}{(s^2+2q_3s+1)(s+2)} \tag{20}$$

where $q_1 \in [0.8, 1.2]$, $q_2 \in [1.6, 2.4]$, and $q_3 \in [0.5, 0.7]$. We also have fixed $y_f = 1$, $\bar{S} = 5\%$, $\bar{U} = 3\%$ and $u_{sat} = 2$.

The nominal transfer function $P_0(s)$ is chosen as

$$P_0(s) = \frac{(s-1)(s-2)}{(s^2+2\cdot 0.6s+1)(s+2)} \tag{21}$$

while eight P_i's transfer functions have been selected by considering the vertexes of the uncertainty space.

The Bode plots of the magnitude of $\Delta_i(j\omega)$, $i = 1,\dots,8$ and that of the selected $W_s(j\omega)$ are reported in Figure 2. It results [1]

$$W_s(s) = \frac{0.05s^3+1.01s^2+1.82s+1.48}{s^3+4.27s^2+3.83s+3.18}. \tag{22}$$

The result of the genetic algorithm is $K = 0.094$, $a = 4.28$ and $\tau = 6.21$ s. As it has been fixed $\varepsilon_0 = 10^{-3}$ and $\varepsilon_1 = 10^{-4}$, the pre-action time results to be $t_p = -5.67$ s, whilst the post-action time is $t_f = 22.0$ s. The determined command input function is plotted, together with the worst-case system output and the corresponding control variable in Figure 3 (for convenience the zero time has been shifted to t_p). The worst-case settling time is $t_{s,wc} = 15.99$ s. It is achieved for the system output obtained when $q_1 = 0.8$, $q_2 = 1.6$ and $q_3 = 0.5$. It can be observed that the active constraint in this case is the one related to the maximum undershoot and that the fixed limit of the control variable is not exceeded during the whole transient.

It is worth stressing that, with respect to the result obtained in (Piazzi and A.Visioli, 2001), where the controller were designed by means of a nominal stable pole-zero cancellation, an improvement of the worst-case settling time has been obtained.

To give a better insight in the results, the Bode plot of the magnitude of the nominal sensitivity function is reported in Figure 4, whilst the complementary sensitivity functions obtained by considering each $P_i(s)$ are plotted in Figure 5. Finally, the normalized power spectrum of the optimal command input function is shown in Figure 6. It appears that the effects of the uncertainty is significantly reduced in the range of frequencies of which the command input function is mainly composed.

4.2 Example 2

As a second example, we consider three eighth-order plants described by the following general transfer function:

$$P(s) = \frac{\sum_{i=0}^{8} a_i s^i}{\sum_{i=0}^{8} b_i s^i} \tag{23}$$

The values of the coefficient for each plant are reported in Table I. Note that the three plants

[1] An expression of W_s can be found with the help of the Matlab μ-Analysis and Synthesis Toolbox function `fitmag`

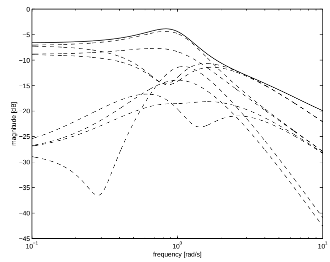

Fig. 2. Bode plot of $|\Delta_i(j\omega)|$ and $|W_s(j\omega)|$ for example 1.

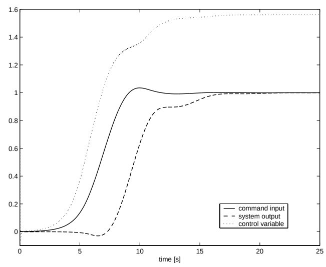

Fig. 3. Optimal command input function, worst-case system output and control variable for example 1.

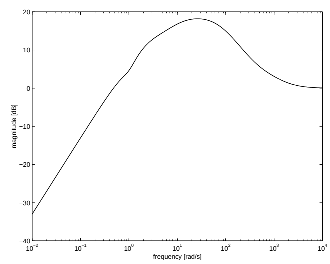

Fig. 4. Bode plot of the resulting nominal sensitivity function for example 1.

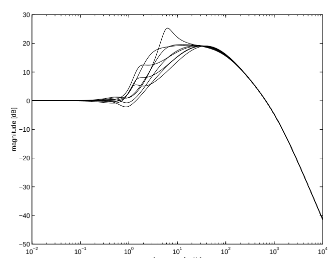

Fig. 5. Bode plot of the resulting complementary sensitivity functions for example 1.

have the same poles but different zeros. We fixed again $y_f = 1$ and the same constraints of the previous example, i.e. $\bar{S} = 5\%$, $\bar{U} = 3\%$ and $u_{sat} = 2$. Further, we select again $\varepsilon_0 = 10^{-3}$ and $\varepsilon_1 = 10^{-4}$.
The nominal transfer function has been selected as

$$P(s) = \frac{-0.1674s^2 + 0.3974s + 10.8}{s^2 + 0.3311s + 9} \qquad (24)$$

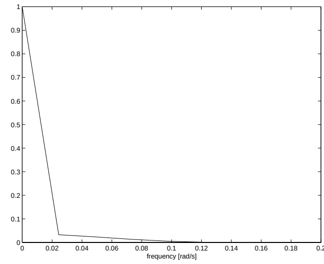

Fig. 6. Normalized power spectrum of the optimal command input function for example 1.

Table 1. Plants for example 2.

Plant	1	2	3
b_0	$9.14 \cdot 10^{10}$	$1.005 \cdot 10^{11}$	$1.097 \cdot 10^{11}$
b_1	$4.294 \cdot 10^{9}$	$4.723 \cdot 10^{9}$	$5.152 \cdot 10^{9}$
b_2	$-1.266 \cdot 10^{9}$	$-1.194 \cdot 10^{9}$	$-1.234 \cdot 10^{9}$
b_3	$-5.756 \cdot 10^{6}$	$-5.951 \cdot 10^{5}$	$-7.922 \cdot 10^{5}$
b_4	$-8.789 \cdot 10^{6}$	$5.109 \cdot 10^{6}$	$2.359 \cdot 10^{6}$
b_5	$-2.675 \cdot 10^{4}$	2785	3785
b_6	-7899	2014	1805
b_7	-6.342	0.08615	1.25
b_8	-0.9848	0.01137	0.2303
a_0	$9.14 \cdot 10^{10}$	$9.14 \cdot 10^{10}$	$9.14 \cdot 10^{10}$
a_1	$4.294 \cdot 10^{9}$	$4.294 \cdot 10^{9}$	$4.294 \cdot 10^{9}$
a_2	$1.049 \cdot 10^{10}$	$1.049 \cdot 10^{10}$	$1.049 \cdot 10^{10}$
a_3	$1.15 \cdot 10^{8}$	$1.15 \cdot 10^{8}$	$1.15 \cdot 10^{8}$
a_4	$3.311 \cdot 10^{7}$	$3.311 \cdot 10^{7}$	$3.311 \cdot 10^{7}$
a_5	$7.849 \cdot 10^{4}$	$7.849 \cdot 10^{4}$	$7.849 \cdot 10^{4}$
a_6	$1.351 \cdot 10^{4}$	$1.351 \cdot 10^{4}$	$1.351 \cdot 10^{4}$
a_7	9.437	9.437	9.437
a_8	1	1	1

whilst the weighting function for the complementary sensitivity function has been chosen as

$$W_s(s) = 160\frac{(s+8)^5}{(s+30)^5}. \qquad (25)$$

The Bode plots of the magnitude of $\Delta_i(j\omega)$, $i = 1, 2, 3$ and that of the selected $W_s(j\omega)$ are reported in Figure 7. The values provided by the genetic algorithm are $K = 0.84$, $a = 0.95$ and $\tau = 0.91$. The resulting command function, which exhibits a preaction time of $t_p = -0.80$ s and a postaction time of $t_f = 2.11$ s is plotted in Figure 8. The resulting worst-case settling time of $t_{s,wc} = 4.16$ s occurs for plant $P_1(s)$, whose output is shown again in Figure 8 as well as the corresponding control variable. As in example 1, we reported the Bode plot of the magnitude of the nominal sensitivity function in Figure 9, whilst the complementary sensitivity functions obtained by considering each $P_i(s)$ are plotted in Figure 10. The normalized power spectrum of the optimal command input function is shown in Figure 11. Indeed, the same conclusions of example 1 can be drawn also for example 2.

5. CONCLUSIONS

In this paper we have presented a new method for the synthesis of a feedback/feedfoward control strategy in order to achieve high performances in

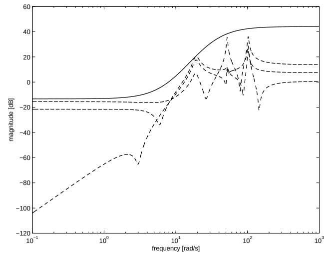

Fig. 7. Bode plot of $|\Delta_i(j\omega)|$ and $|W_s(j\omega)|$ for example 2.

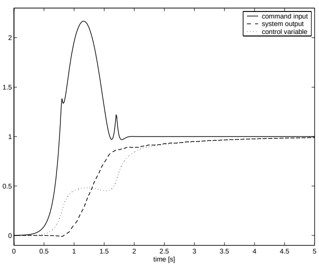

Fig. 8. Optimal command input function, worst-case system output and control variable for example 2.

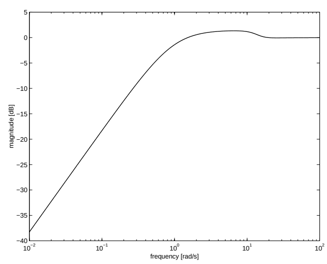

Fig. 9. Bode plot of the resulting nominal sensitivity function for example 2.

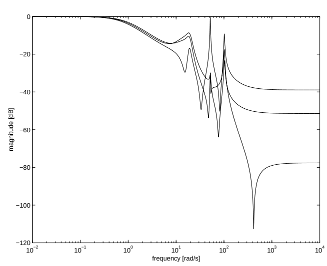

Fig. 10. Bode plot of the resulting complementary sensitivity functions for example 2.

the robust constrained set-point regulation problem. The technique combines appropriately the $\mathcal{H}_\infty$ control and the dynamic inversion concepts, in order to exploit the main useful characteristics of both of them, although the robust stability issue is handled in a conservative way. A salient feature of the design methodology is that it can be performed automatically with available software packages.

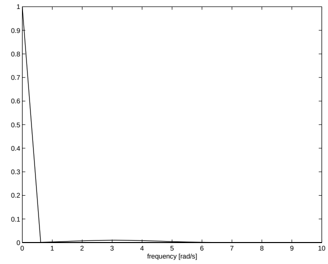

Fig. 11. Normalized power spectrum of the optimal command input function for example 2.

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