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An Iterative Approach for Noncausal Feedforward Tuning

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***Abstract*— A new iterative approach for the determination of a noncausal feedforward control law suitable for set-point regulation is presented in this paper. The technique aims at estimating recursively the parameters of the system to be controlled in order to determine the exact noncausal command input to be applied to the closed-loop system in order to achieve a predefined output transition. In this context, a gradient based minimization of the integrated square error cost function is performed. Simulation results show the effectiveness of the methodology for different system characteristics.**

I. INTRODUCTION

It is well known that a properly designed feedforward controller is capable to improve significantly the performance of a feedback control system for set-point regulation. When preview information is available so that the next desired constant output is known in advance, a noncausal feedforward controller may be adopted. This corresponds to the synthesis of a command input with pre- and post-actuation (in the general case) by using stable dynamic inversion procedures [1]-[11]. Indeed, the problem consists in finding the suitable command signal that has to be applied to the (closed-loop) system in order to achieve a predefined transient response when the process output is required to assume a new value. In particular, an analytical solution to the problem has been provided in [13] when the selected desired output function is a polynomial function [12].

In this context, despite the fact that a feedback controller provides robustness to the control system with respect to parameter variations (in addition to compensate for external disturbances), it is obvious that the performance obtained depends on the accuracy of the system model [14].

In this paper we propose an iterative methodology for the estimation of the parameters of the system in order to minimize the integrated square error cost function, where the error considered is the difference between the obtained system transient response and the one that is desired. A gradient-based minimization is used for this purpose. The approach is inspired by the well-known Iterative Feedback Tuning (IFT) [15], [16] methodology, which aims at finding optimal causal feedforward/feedback controller parameters and which has been proven to be effective both from a theoretical and a practical point of view. In the proposed technique the structure of the model is assumed to be known and therefore an additional experiment does not need to be performed (at each iteration) in order to generate the gradient expression.

It is worth stressing that the closed-loop controller does not change during the application of the method and therefore the disturbance rejection performance of the control system are not modified. In any case, the obtained process model can be eventually employed in order to redesign the controller or to assess its performance.

Notation. C^i denotes the set of scalar real functions that are continuous till the ith derivative and BC^i denotes the subset of C^i of the scalar real functions that are bounded over $\mathbb{R}$.

II. ITERATIVE NONCAUSAL FEEDFORWARD TUNING

A. Problem formulation

Consider the standard unity-feedback system shown in Figure 1 where $\hat{P}(s;\rho)$ is an estimated scalar linear time-invariant rational transfer function of the (unknown) "true" system $P(s)$ to be controlled, parameterized by a parameter vector $\rho \in \mathbb{R}^{n_\rho}$, $C(s)$ is the controller transfer function and r is a command input signal to be applied to the closed-loop control system.

The (estimated) closed-loop transfer function is denoted as:

$$\hat{T}(s;\rho) := \frac{C(s)\hat{P}(s;\rho)}{1 + C(s)\hat{P}(s;\rho)}, \tag{1}$$

while the (unknown) "true" closed-loop transfer function is denoted as:

$$T(s) := \frac{C(s)P(s)}{1 + C(s)P(s)}, \tag{2}$$

The set of all cause/effect pairs $(r(\cdot), y(\cdot))$ associated with the closed-loop system is denoted by $\mathcal{B}$. In the framework of the behavioral approach, $\mathcal{B}$ is the behavior set of the system that can be rigorously introduced by means of the so-called *weak solutions* of the differential equation associated to the system [17].

The considered regulation problem consists of obtaining an output transition from a previous value y_0 to a new value y_1. Without loss of generality, in the following we will consider $y_0 = 0$. The relative order of the closed-loop system is known and denoted as μ.

Define $y_d(\cdot) \in BC^k$ with $y_d(t) = 0$ for $t < 0$ as the desired output function to obtain the transition. From a practical point of view, a transition time τ has to be defined, *i.e.*, the desired output function is defined as

$$y_d(t) := \begin{cases} 0 & \text{for} \quad t < 0 \\ y_{01}(t) & \text{for} \quad 0 \le t \le \tau \\ y_1 & \text{for} \quad t > \tau \end{cases} \tag{3}$$

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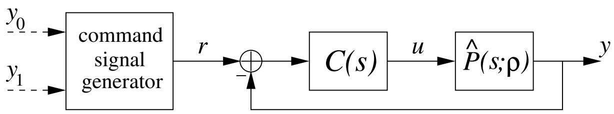

Fig. 1. The unity-feedback control scheme.

where $y_{01}(t)$ is the desired transition function between the constant values 0 and y_1. The stable input-output inversion problem consists in determining $r(\cdot) \in BC^{k-\mu}$ such that the desired output function is obtained, *i.e.*, such that

$$(r(\cdot), y_d(\cdot)) \in \mathcal{B}. \tag{4}$$

The general solution to this problem, which involves a numerical computation, can be found in [11]. It is worth stressing that when the selected desired output function is a polynomial function [12], an analytical solution can be found [13] and this fact can be exploited in speeding up the computational time and most of all in avoiding numerical problems.

In general, $r(t)$ is defined over the time interval $(-\infty, +\infty)$ and therefore, in order to practically use it, it is necessary to truncate it. Thus, the input function exhibits a pre-actuation (associated with the unstable zeros) and a post-actuation (associated with the stable zeros) time intervals (see for example [18]).

The stable input-output procedure has necessarily to be applied to estimated known closed-loop transfer function $\hat{T}(s;\rho)$, but, obviously, considered that the true transfer function is $T(s)$, which may encompass unstructured uncertainties, then the desired output function is not actually obtained, although the presence of the feedback controller reduces the detrimental effects of the uncertainties [5]. In other words, by applying the determined command input $r(t;\rho)$ to the true system, the obtained system output $y(t;\rho)$ is not equal to $y_d(t)$. For this reason, an iterative strategy is proposed in order to find the optimal parameter vector defined by

$$\rho^* := \arg\min_{\rho} J(\rho) \tag{5}$$

where $J(\rho)$ is the following integral criterion [19], [20]:

$$J(\rho) = \frac{1}{2T_f}\int_0^{T_f} (y(t;\rho) - y_d(t))^2 dt \tag{6}$$

where T_f is the control interval (which might be practically selected as the sum of the pre- and post-actuation time intervals and of the transition time τ).

B. Generating the cost function gradient

In order to solve the optimization problem (5), a gradient-based minimization can be adopted, namely, the following expression can be used after the ith iteration:

$$\rho(i+1) = \rho(i) - \gamma_i R_i^{-1}(i)\frac{\mathrm{d}J}{\mathrm{d}\rho}(\rho(i)) \tag{7}$$

where γ_i is a positive real scalar that determines the step size and R_i is some appropriate positive definite matrix.

The computation of the cost function gradient is therefore necessary in order to apply the gradient search (7). From (6) it follows that

$$\frac{\mathrm{d}J}{\mathrm{d}\rho}(\rho(i)) = \frac{1}{T_f}\int_0^{T_f} e(t;\rho(i))\frac{\mathrm{d}e(t;\rho(i))}{\mathrm{d}\rho}dt \tag{8}$$

where

$$e(t;\rho) = y(t;\rho) - y_d(t). \tag{9}$$

It can be noted that $e(t;\rho(i))$ is immediately available after an experiment has been performed. In order to determine an expression of the first derivative of the error signal, since an analytical expression cannot be derived in general, it is convenient to consider the Laplace transform of the functions considered, namely,

$$\begin{aligned} E(s;\rho) = \quad & Y(s;\rho) - Y_d(s) = \\ & T(s)R(s;\rho) - Y_d(s). \end{aligned} \tag{10}$$

By considering that $R(s;\rho)$ has been obtained by inverting the system $\hat{T}(s;\rho)$, we can write

$$\begin{aligned} R(s;\rho) = \quad & \hat{T}^{-1}(s;\rho)Y_d(s) = \\ & \left[\frac{1 + C(s)\hat{P}(s;\rho)}{C(s)\hat{P}(s;\rho)}\right] Y_d(s) \end{aligned} \tag{11}$$

and, hence, we obtain:

$$\frac{\mathrm{d}E(s;\rho)}{\mathrm{d}\rho} = -T(s)Y_d(s)\frac{\frac{d\hat{P}(s;\rho)}{d\rho}}{C(s)\hat{P}^2(s;\rho)}. \tag{12}$$

Then, by multiplying both the numerator and the denominator by $1 + C(s)\hat{P}(s;\rho)$, we have

$$\begin{aligned} \frac{\mathrm{d}E(s;\rho)}{\mathrm{d}\rho} = \quad & -\frac{d\hat{P}(s;\rho)}{d\rho}\frac{1}{1 + C(s)\hat{P}(s;\rho)}\cdot \\ & \cdot\hat{P}^{-1}(s;\rho)T(s)\hat{T}^{-1}(s;\rho)Y_d(s) = \\ & -\frac{d\hat{P}(s;\rho)}{d\rho}\frac{1}{1 + C(s)\hat{P}(s;\rho)}\hat{P}^{-1}(s;\rho)Y(s;\rho) \end{aligned} \tag{13}$$

Thus, by considering the corresponding time-domain signals, the term $\mathrm{d}e(t;\rho)/\mathrm{d}\rho$ can be computed in principle by determining the response of the system $-(\mathrm{d}\hat{P}(s;\rho)/\mathrm{d}\rho)\cdot\hat{P}^{-1}(s;\rho)/(1+C(s)\hat{P}(s;\rho))$ to $y(t;\rho)$. However, this is not possible if $\hat{P}(s;\rho)$ is nonminimum-phase, since $\hat{P}^{-1}(s;\rho)$ is unstable in this case. Hence, in general, the gradient of the error signal has to be calculated by first determining the signal $\hat{u}(t;\rho)$ which, applied to the system $\hat{P}(s;\rho)$, causes the collected system output $y(t;\rho)$ (*i.e.*, by applying the stable input-output procedure to the system $\hat{P}(s;\rho)$ with a desired system output $y(t;\rho)$) and then by determining the response of the system $-(\mathrm{d}\hat{P}(s;\rho)/\mathrm{d}\rho)/(1+C(s)\hat{P}(s;\rho))$ to $\hat{u}(t;\rho)$.

C. Algorithm

Based on the above considerations, given an estimated system transfer function $\hat{P}(s;\rho)$, a controller transfer function $C(s)$ and a desired output function $y_d(t)$, the overall Iterative Noncausal Feedforward Tuning algorithm can be posed as follows.

INFT algorithm

1) Determine the command input function $r(t;\rho)$ by applying a stable input-output inversion procedure to the closed-loop system $\hat{T}(s;\rho)$ with output function $y_d(t)$.
2) Run a closed-loop system experiment with command input $r(t;\rho)$.
3) Record the system output $y(t;\rho)$ and determine the error signal $e(t;\rho) = y(t;\rho) - y_d(t)$.
4) Determine (by applying a stable input-output inversion procedure) the signal $\hat{u}(t;\rho)$ that, applied to the system $\hat{P}(s;\rho)$, causes the output signal $y(t;\rho)$.
5) Determine $\mathrm{d}e(t;\rho)/\mathrm{d}\rho$ as the response of the system
$$-\frac{\mathrm{d}\hat{P}(s;\rho)}{\mathrm{d}\rho}\frac{1}{1+C(s)\hat{P}(s;\rho)}$$
to the signal $\hat{u}(t;\rho)$.
6) Calculate the cost function gradient (8).
7) If $\|\mathrm{d}J(\rho(i))/\mathrm{d}\rho\| > \varepsilon$ then update the parameter vector ρ by applying formula (7) and go to 1, else terminate.

Note that, although the algorithm converges to the set of stationary points of the criterion (6) (see subsection III), the termination condition at step 7 is employed in order to avoid a infinite number of iterations. In this context the user-chosen parameter ε allows to handle the trade-off between the number of iterations and the achieved performance.

Remark 1. It is worth stressing that the process model structure is assumed to be known and therefore the methodology is particularly suitable for systems with structured (parametric) uncertainties. However, it has to be considered that the presence of the feedback controller reduces the effects of the (unstructured) uncertainties at low frequencies where the determined command input has its frequency content [21]. Thus, the approach is effective even in the presence of unstructured uncertainties (see Section IV).

Remark 2. With respect to the classical deterministic version of the IFT method [20], here the considered cost function does not include the term related to the control effort (as in [19]). This is due to the fact that the trade-off between aggressiveness of the control system and its control effort can be easily handled by the choice of the transition time τ (see (3)) [13], [10].

Remark 3. Note that, in order to provide a useful tool for the application of a stable input-output inversion procedure (step 1 and 4 of the algorithm), a Matlab toolbox has been implemented [22].

III. PRACTICAL ISSUES

It is worth stressing that, since all the adopted signals are bounded, the convergence properties of the algorithm can be derived by following an analysis analogous to the one of the IFT methodology [15], [16]. Further, the considerations made in [15], [16] regarding the choice of the sequence of matrices R_i in (7) apply also for the INFT technique.

It is also worth stressing at this point that the presence of measurement noise (which has not been taken into account explicitly in the previous analysis) can be addressed by an appropriate filtering of the data. Note that a standard off-line filtering technique can be employed.

IV. SIMULATION RESULT

As a first illustrative example, consider the following system with a positive real zero:

$$P(s) = \frac{-s+1}{(2s+1)(3s+1)}. \tag{14}$$

Assume that an initial estimated model is

$$\hat{P}(s;\rho) = \frac{zs+1}{(T_1 s+1)(T_2 s+1)} \tag{15}$$

where $\rho(1) = [z,\ T_1,\ T_2] = [-1.5,\ 4,\ 1]$ is the initial parameter vector. The controller is chosen as a PID controller whose transfer function is

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1} \tag{16}$$

where the value of the parameters has been selected as $K_p = 1.56$, $T_i = 5.03$, $T_d = 1.26$, $T_f = 0.01$. The desired output function is chosen as the following transition polynomial [12]

$$y_d(t) := \begin{cases} 0 & \text{for} \quad t < 0 \\ -2\dfrac{t^3}{\tau^3} + 3\dfrac{t^2}{\tau^2} & \text{for} \quad 0 \le t \le \tau \\ 1 & \text{for} \quad t > \tau \end{cases} \tag{17}$$

where $\tau = 2$.

The application of the command input determined by applying the stable input-output procedure to the closed-loop system provides the system output shown in Figure 2, where it is compared with the desired output transition function (17). Note that the resulting pre-actuation time is equal to -10.36 and the post-actuation time is 3.19. After running the INFT procedure with $\gamma = 0.07$ and R equal to the identity matrix, at the tenth iteration we have $\rho(10) = [-1.08,\ 3.92,\ 1.45]$. The resulting system output at the tenth iteration is shown in Figure 3 (in this case the pre-actuation time is equal to -7.79 and the post-actuation time is 1.83), where it is again compared with the desired output transition function. The command input determined at the first and at the tenth iteration are shown in Figure 4, while the value of the cost function $J(\rho)$ during the execution of the INFT procedure are shown in Figure 5. It can be seen that the procedure allows to significantly improve the

performance in a few number of iterations.
As a second example, consider the system

$$P(s) = \frac{1}{(s+1)^3}e^{-s}. \tag{18}$$

The initial estimated model is a first-order-plus-dead-time transfer function

$$\hat{P}(s;\rho) = \frac{K}{Ts+1}e^{-Ls} \tag{19}$$

where $\rho(1) = [K,\ T,\ L] = [0.86,\ 2.20,\ 2]$ is the initial parameter vector. A PID controller (16) with $K_p = 1$, $T_i = 4.37$, $T_d = 1.09$, $T_f = 0.01$ has been chosen as a controller and function (17) with $\tau = 3$ has been chosen as a desired output transition function. The INFT procedure is then run with $\gamma = 0.006$ and R equal to the identity matrix. The system output resulting at the first and tenth iteration (together with the corresponding desired output functions) are shown in Figures 6 and 7 respectively (the pre-actuation time is -6.67 at the first iteration and -7.04 at the tenth one, while the post-actuation time is 21.85 in both cases). Note that $\rho(10) = [1.01,\ 2.29,\ 2.12]$. The command input determined at the first and at the tenth iteration are shown in Figure 8, while the value of the cost function $J(\rho)$ during the execution of the INFT procedure are shown in Figure 9. Consider now, as a third example, the same previous example but with a different initial parameter vector $\rho(1) = [K,\ T,\ L] = [0.86,\ 4.0,\ 2.20]$ (*i.e.*, the estimated time constant and the estimated dead time are increased). The results related to the first and the hundredth iteration ($\gamma = 0.01$) are shown in Figures 10-12 and the value of the cost function is shown in Figure 13. Note that $\rho(30) = [0.97,\ 1.91,\ 2.27]$ and that a significant improvement of the performance is achieved after a few iterations, despite the results related to many of them are shown. It turns out that the methodology is capable to improve the performance significantly despite a different initial estimation of the parameters.

V. CONCLUSIONS

In this paper we have presented an iterative approach for the tuning of a noncausal feedforward action based on a stable input-output inversion procedure. The methodology allows to estimate the parameters of the system model in order to minimize the integrated square error. The obtained system model can then be adopted also for the purpose of tuning the feedback controller. It has to be remarked that in any case the feedback controller does not change during the iterative procedure and therefore the disturbance rejection performance of the control system is not affected. In other words, the set-point following performance of the control system are addressed independently of the disturbance rejection performance. Simulation results have demonstrated that a small number of iterations suffices for a significant improvement of the regulation performance.

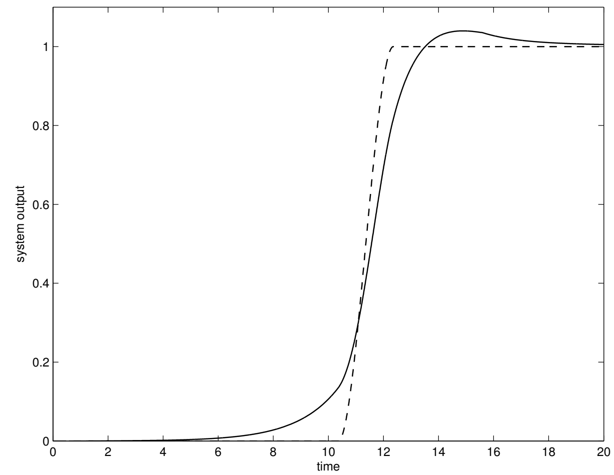

Fig. 2. Obtained (solid line) and desired (dashed line) system output before applying the INFT procedure for example 1.

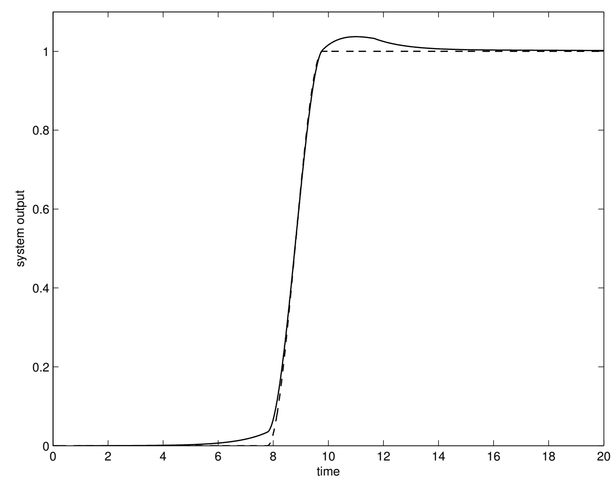

Fig. 3. Obtained (solid line) and desired (dashed line) system output at the tenth iteration of the INFT procedure for example 1.

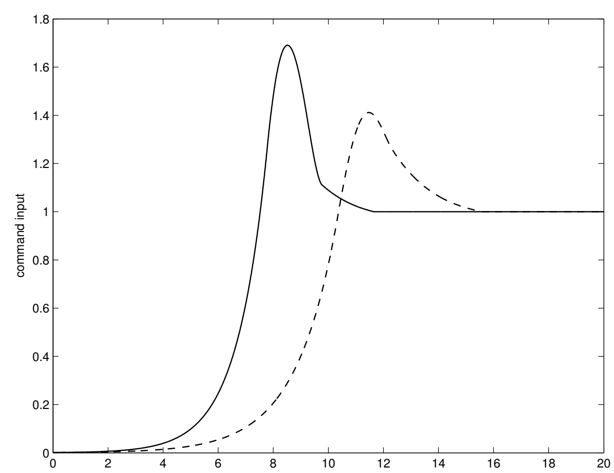

Fig. 4. Closed-loop command input determined at the first (dashed line) and at the tenth iteration (solid line) of the INFT procedure for example 1.

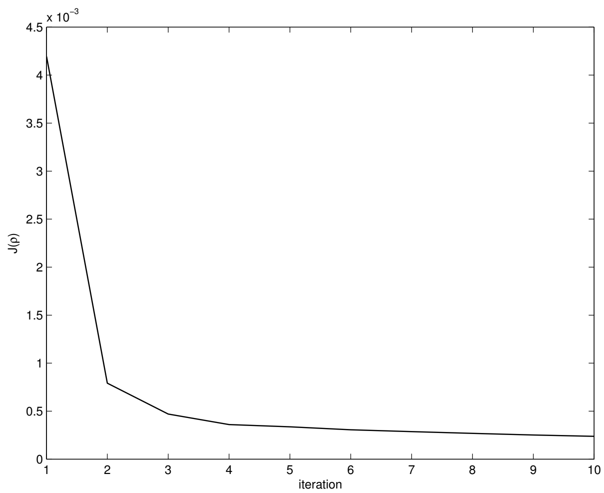

Fig. 5. Cost function for example 1.

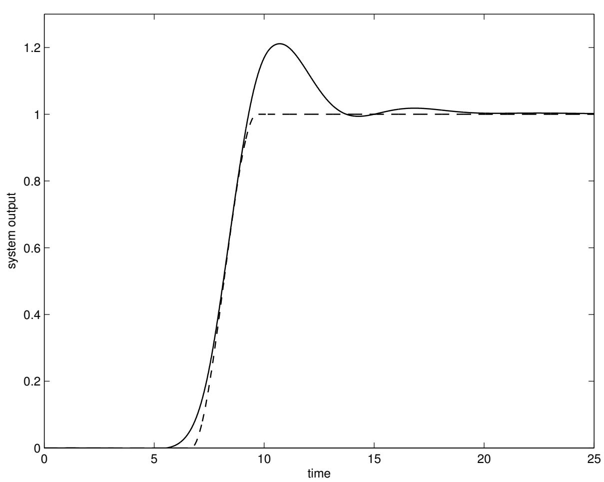

Fig. 6. Obtained (solid line) and desired (dashed line) system output before applying the INFT procedure for example 2.

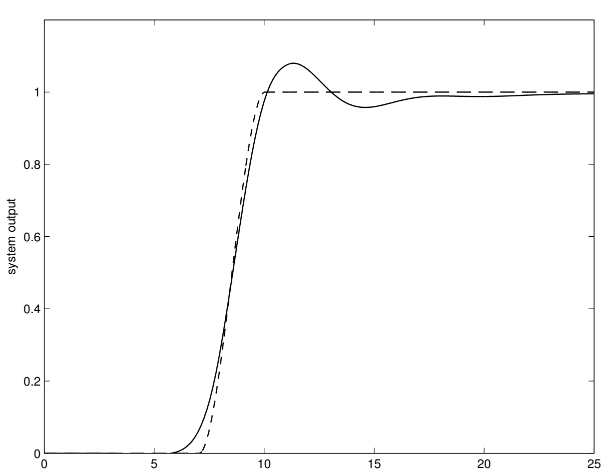

Fig. 7. Obtained (solid line) and desired (dashed line) system output at the tenth iteration of the INFT procedure for example 2.

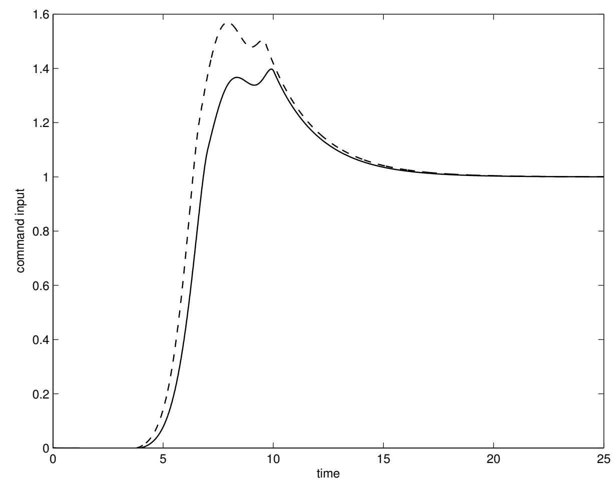

Fig. 8. Closed-loop command input determined at the first (dashed line) and at the tenth iteration (solid line) of the INFT procedure for example 2.

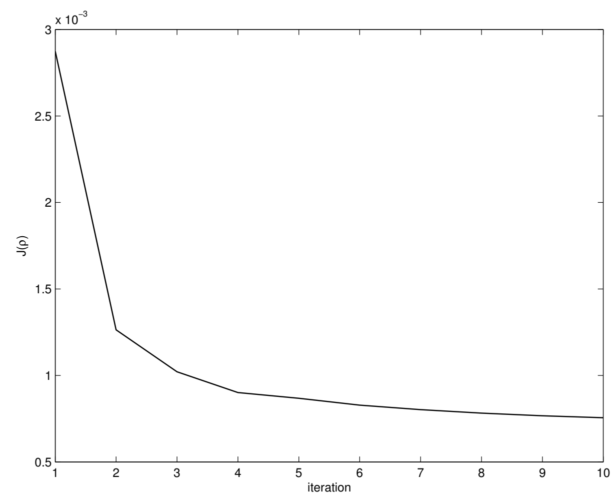

Fig. 9. Cost function for example 2.

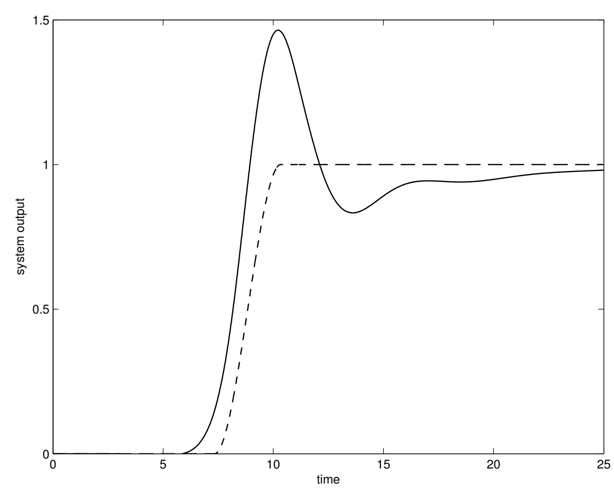

Fig. 10. Obtained (solid line) and desired (dashed line) system output before applying the INFT procedure for example 3.

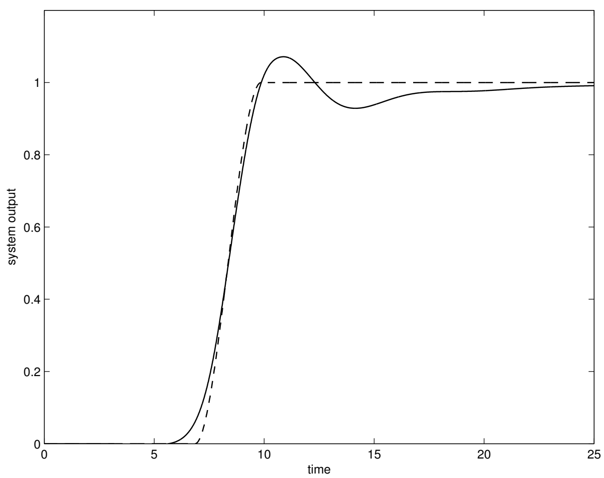

Fig. 11. Obtained (solid line) and desired (dashed line) system output at the tenth iteration of the INFT procedure for example 3.

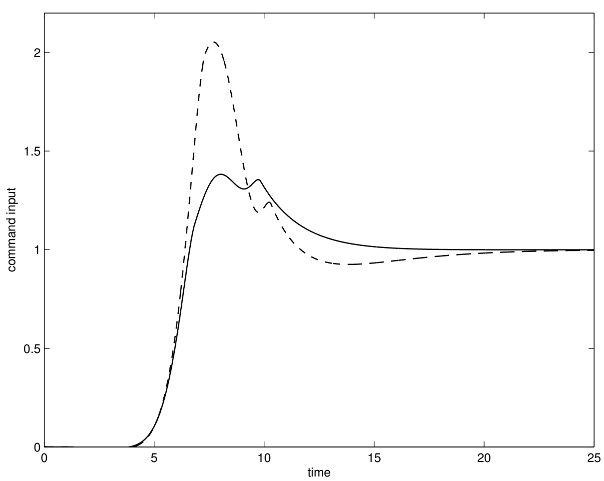

Fig. 12. Closed-loop command input determined at the first (dashed line) and at the tenth iteration (solid line) of the INFT procedure for example 3.

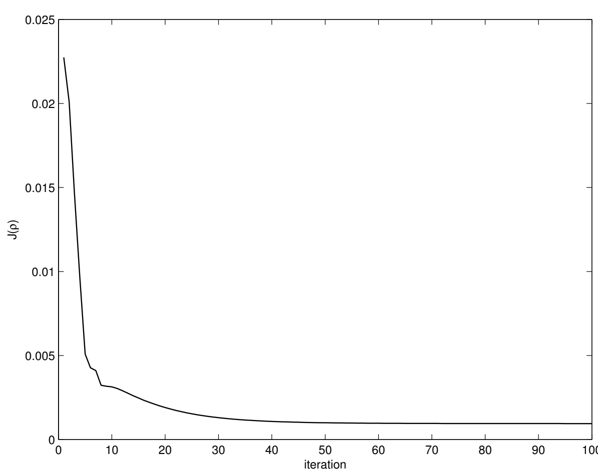

Fig. 13. Cost function for example 3.

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