Proceedings of the European Control Conference 2007
Kos, Greece, July 2-5, 2007

Minimum-time feedforward control for industrial processes

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Abstract— **In this paper we present a design methodology for Proportional-Integral-Derivative (PID) control loops in order to obtain a minimum-time process output transition subject to constraints on the process variable and on the manipulated variable. The technique relies on the synthesis of a suitable command input that has to be applied to the closed-loop system, where the PID controller has been tuned previously by any conventional method. Simulation as well as experimental results related to a level control problem in a laboratory setup are shown to demonstrate the effectiveness of the approach.**

I. INTRODUCTION

Proportional–Integral–Derivative (PID) controllers are undoubtedly the most widely adopted controllers in industry owing to the advantageous cost/benefit ratio they are able to provide. In order to help the operator to select the controller gains to address given control specifications, many tuning formulas have been devised in the past [1] and autotuning functionalities are almost always available in commercial products [2], [3].

However, it is also recognized that the performance of a PID control loop is determined also by the suitable implementation of those functionalities that have to (or can) be added to the basic PID control law [4]. In this context a particular attention has been paid by researchers to the synthesis of a suitable feedforward control action in order to improve the set-point following performance, especially when the tuning of the PID parameters is devoted to the load disturbance rejection performance. The use of the well-known set-point weighting strategy [5] falls in this framework. The main disadvantage of this method is that the reduction of the overshoot is paid by a slower set-point response. To overcome this drawback, the use of a variable set-point weight [6], [7] has been proposed. With the aim of improving the classical linear feedforward controller design [3], [8], a nonlinear feedforward action has been proposed in [9], [10], [11], where the constraints on the actuators are considered explicitly. From another point of view, a noncausal approach has been developed in [12]. In this case, the choice of the desired process variable transition time can be exploited to handle the trade-off between aggressiveness and robustness and control effort.

However, in all the mentioned approaches, constraints on both the actuators and the process variable are not considered explicitly. In this paper we propose the application of a new minimum-time feedforward control strategy to a PID control loop where both actuator limits as well as constraints on the maximum overshoot and undershoot are taken into account, making the problem very appealing from a practical point of view. The command input is synthesized by means of a linear programming approach. In fact after discretization the minimum-time control problem can be solved with a sequence of feasibility tests of a LP-problem. A distinguished feature of this method is that both the plant and the controller reach the equilibrium at the end of the transition time, therefore all the system signals are constant and there are not remaining decaying zero-dynamics in the plant input or in the reference signal. The article is divided as follows. The first section introduces the minimum-time feedback problem. The second one presents the proposed algorithm, while the third and the fourth one present some simulations and experiments.

Partial support for this research has been provided by MIUR scientific research funds.

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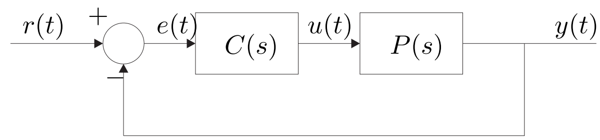

Fig. 1. The control scheme.

II. THE MINIMUM-TIME FEEDFORWARD CONTROL PROBLEM

The feedback control scheme considered in this work is presented in Figure 1. The set point function $r(t)$ is assumed to be a function of time and the problem consists in finding the optimal set-point signal $r(t)$ such that the output $y(t)$ performs a rest-to-rest transition in minimum time from 0 to the final value y_f. The overall control system reaches the equilibrium at the end of the transition time.

It is supposed that the feedback controlled $C(s)$ has already been designed. The closed-loop system Σ is linear and continuous-time and described by the scalar transfer function

$$T_{ry}(s) = \frac{b(s)}{a(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0} \quad , \qquad (1)$$

where $a(s)$ and $b(s)$ are coprime. Σ is stable, i.e. $a(s)$ is an Hurwitz polynomial, and its static gain $T_{ry}(0) = \frac{b_0}{a_0} \neq 0$. The behavior set of Σ can be introduced as the set $\mathcal{B}$ of all input-output pairs $(r(\cdot), y(\cdot)) \in L^1_{\text{loc}} \times L^1_{\text{loc}}$ that are "weak" solutions of the differential equation [13]:

$$\sum_{i=0}^{n} a_i D^i y = \sum_{i=0}^{m} b_i D^i r \quad . \qquad (2)$$

ISBN: 978-960-89028-5-5

The set of input-output equilibrium points of Σ is denoted by $\mathcal{E} := \{(r,y) \in \mathbb{R}^2 : y = T_{ry}(0)r\}$. We introduce, as a special subset of $\mathcal{B}$, the set $\mathcal{T}_\mathbf{p}$ of all rest-to-rest transitions from $(0,0) \in \mathcal{E}$ to $(\frac{y_f}{T_{ry}(0)}, y_f) \in \mathcal{E}$ subject to given constraints on the input r and the output y.

Definition 1: Given the parameter set $\mathbf{p} := \{U_c, Y_c, y_f\}$ where $U_c = [u_c^-, u_c^+]$ and $Y_c = [y_c^-, y_c^+]$ are the constraint intervals for the input and output respectively and y_f is the final rest value of output, $\mathcal{T}_\mathbf{p}$ is the set of all $(r(\cdot), y(\cdot)) \in \mathcal{B}$ for which there exists $t_f > 0$ such that:

$$r(t) = 0 \ \forall t < 0 \ , r(t) = \tfrac{y_f}{T_{ry}(0)} \ \forall t \geq t_f \ , \quad (3)$$
$$u(t) = 0 \ \forall t < 0 \ , u(t) = \tfrac{y_f}{P(0)} \ \forall t \geq t_f \ , \quad (4)$$
$$u(t) \in U_c \ \ \forall t \in \mathbb{R} \ , \quad (5)$$
$$y(t) = 0 \ \forall t < 0 \ , y(t) = y_f \ \ \forall t > t_f \ , \quad (6)$$
$$y(t) \in Y_c \ \forall t \in \mathbb{R} \ . \quad (7)$$

The constraints intervals introduced in the above definition can encapsulate all the typical amplitude restrictions that apply to the input and the output. For example, if a given control application requires $|u(t)| \leq u_{MAX} \ \forall t \in \mathbb{R}$, a maximum 10% overshooting and 5% undershooting on the output signal we can assign (consider $y_f > 0$)

$$U_c = [-u_{MAX}, \ +u_{MAX}] \ , \ \ Y_c = [-0.05 y_f, \ +1.1 y_f] \ .$$

The following theorem gives a straightforward sufficient condition to ensure that $\mathcal{T}_\mathbf{p}$ is not empty.

Theorem 1: Set $\mathcal{T}_\mathbf{p}$ is not empty if

$$\{0, \frac{y_f}{P(0)}\} \subset (u_c^-, u_c^+) \quad \text{and} \quad \{0, y_f\} \subset (y_c^-, y_c^+) \ . \quad (8)$$

and if there are not cancellations beween poles of $C(s)$ and zeros of $P(s)$.

The proof is a straightforward extension of an analogous result presented in [14]. It is based on the idea that when the input rate of variation tends to zero, the output tends to be equal to the input multiplied by the system static gain.

Remark 1: The result presented in Theorem 1 holds also for systems with time-delays.

Remark 2: Even though the above theorem provides only a sufficient condition it is worth noting that inclusions (8) are almost necessary. Indeed if $0 \notin U_c \vee \frac{y_f}{P(0)} \notin U_c \vee 0 \notin Y_c \vee y_f \notin Y_c$ then $\mathcal{T}_\mathbf{p}$ is evidently empty because of the incompatibility of the input-output rest values with the required constraints.

Once inclusions (8) are satisfied, the emerging natural problem is to determine among all the constrained transitions of $\mathcal{T}_\mathbf{p}$ the fastest one, i.e. the optimal rest-to-rest transition with associated minimum transition time t_f^*:

$$t_f^* := \inf_{(r(\cdot),y(\cdot)) \in \mathcal{T}_\mathbf{p}} T_f(r(\cdot), y(\cdot)) \quad (9)$$

where T_f is the following functional

$$T_f(r(\cdot), y(\cdot)) = \inf\{t_1 : r(t) = \frac{y_f}{T_{ry}(0)}, \ y(t) = y_f \ , \forall t \geq t_1\} \quad (10)$$

which is well defined by Definition 1.

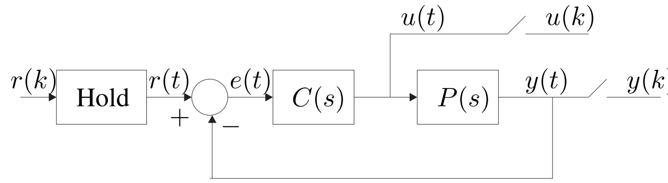

Fig. 2. The discrete control scheme.

III. ALGORITHM

In this section, the minimum-time feedforward control problem is approximated with a discrete-time one and a solution is provided using linear programming.

The continuous control problem can be approximated with a discrete one as follows. Consider the scheme represented in Figure 2. This system corresponds to the one represented in Figure 1, with the difference that the command function $r(t)$ is assumed to be given by a piecewise control signal obtained from a discrete time one through a zero-order hold. Discrete-time signals $u(k)$ and $y(k)$ are obtained by sampling the continuous input and output $u(t)$ and $y(t)$, with sampling time T.

Let Σ_d be the closed-loop discrete system, then the transfer function from discrete-time input $r(k)$ to the sampled output $y(k)$ is given by

$$T_{ry}(z) = \frac{b(z)}{a(z)} = \frac{b_m z^m + b_{m-1} z^{m-1} + \cdots + b_0}{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0} \ . \quad (11)$$

Σ_d is stable, and its static gain $T_{ry}(1) \neq 0$. The system input and output sequences are denoted by $r(k)$ and $y(k)$ respectively, $k \in \mathbb{Z}$.

The behavior $\mathcal{B}_d$ of system Σ_d is the set of all input-output pairs $(r(\cdot), y(\cdot))$, where $r(\cdot), y(\cdot) : \mathbb{Z} \to \mathbb{R}$, satisfying the difference equation:

$$a_n r(k+n) + a_{n-1} r(k+n-1) + \cdots + a_0 r(k) = b_m u(k+m) + b_{m-1} u(k+m-1) + \cdots + b_0 u(k) \ . \quad (12)$$

The set of input-output equilibrium points of Σ_d is $\mathcal{E} := \{(r,y) \in \mathbb{R}^2 : y = T_{ry}(1)u\}$ and the set $\mathcal{K}_\mathbf{p} \subset \mathcal{B}_d$ of all rest-to-rest constrained transitions from $(0,0) \in \mathcal{E}$ to $(\frac{y_f}{T_{ry}(1)}, y_f) \in \mathcal{E}$ is defined as follows.

Definition 2: Given the parameter set $\mathbf{p} := \{U_c, Y_c, y_f\}$ where $U_c = [u_c^-, u_c^+]$ and $Y_c = [y_c^-, y_c^+]$ are the constraint intervals for the input and output respectively and y_f is the final rest value of the output, $\mathcal{K}_\mathbf{p}$ is the set of all pairs $(r(\cdot), y(\cdot)) \in \mathcal{B}_d$ for which there exists $k_f \in \mathbb{N}$ such that:

$$r(k) = 0 \ \forall k < 0 \ , \ r(k) = \tfrac{y_f}{T_{ry}(1)} \ \forall k \geq k_f \ , \quad (13)$$
$$u(k) = 0, \ \forall k < 0 \ , u(k) = \tfrac{y_f}{P(0)}, \ \forall k \geq k_f \ , \quad (14)$$
$$u(k) \in U_c . \ \forall k \in \mathbb{Z} \ , \quad (15)$$
$$y(k) = 0 \ \ \forall k < 0 \ , \quad y(k) = y_f \ \ \forall k \geq k_f \ , \quad (16)$$
$$y(k) \in Y_c \ \ \forall k \in \mathbb{Z} \ . \quad (17)$$

The following result is the discrete counterpart of Theorem 1.

Theorem 2: Set $\mathcal{K}_{\mathbf{p}}$ is not empty if

$$\{0, \frac{y_f}{P(1)}\} \subset (u_c^-, u_c^+) \quad \text{and} \quad \{0, y_f\} \subset (y_c^-, y_c^+) \ . \quad (18)$$

and if there are not cancellations between poles of $C(s)$ and zeros of $P(s)$.

*Proof.*Omitted for brevity.

The minimum-time feedforward constrained control problem for discrete-time systems consists in finding the optimal input sequence $r^*(k)$, $k = 0, 1, \dots, k_f^* - 1$ for which the pair $(r^*(\cdot), y^*(\cdot)) \in \mathcal{K}_{\mathbf{p}}$ is a minimizer for the optimization problem:

$$k_f^* = \min_{(r(\cdot),y(\cdot))\in\mathcal{K}_{\mathbf{p}}} K_f(r(\cdot), y(\cdot)) \ . \quad (19)$$

$K_f(r(\cdot), y(\cdot))$, the rest-to-rest transition time associated to pair $(r(\cdot), y(\cdot))$, is defined as follows

$$K_f(r(\cdot), y(\cdot)) :=$$
$$\min\{k_1 \in \mathbb{N} : r(k) = \frac{y_f}{T_{ry}(1)},\ y(k) = y_f\ , \forall k \geq k_1\} \ .$$

The key result upon which to build the solution to the minimum-time problem is given by the next proposition. The closed-loop transfer function T_{ry} can be determined as $T_{ry} = (1 - z^{-1})\mathcal{Z}\{\frac{C(s)P(s)}{1+C(s)P(s)}\}$, while $T_{ru} = (1 - z^{-1})\mathcal{Z}\{\frac{C(s)}{1+C(s)P(s)}\}$ represents the discrete transfer function from the reference $r(k)$ to the sampled plant input $u(k)$. Let the unit impulse response of T_{ry} be denoted by $h_{ry}(k) := \mathcal{Z}^{-1}[T_{ry}(z)]$, the unit impulse response of T_{ru} be denoted by $h_{ru}(k) := \mathcal{Z}^{-1}[T_{ru}(z)]$ and let $\mathbf{1}_k$ denote the k-dimensional vector whose components are all equal to 1.

Proposition 1: Set $\mathcal{K}_{\mathbf{p}}$ is not empty if if and only if there exist $k_f \in \mathbb{N}$ and a vector $\mathbf{r} \in \mathbb{R}^{k_f}$ for which the following LP problem is feasible:

$$y_c^- \cdot \mathbf{1}_{k_f} \leq \mathbf{T}^{ry}\mathbf{r} \leq y_c^+ \cdot \mathbf{1}_{k_f} \quad (20)$$

$$u_c^- \cdot \mathbf{1}_{k_f} \leq \mathbf{T}^{ru}\mathbf{r} \leq u_c^+ \cdot \mathbf{1}_{k_f} \quad (21)$$

$$\bar{\mathbf{T}}^{ry} \left[\begin{array}{c} \mathbf{r} \\ \hline \dfrac{y_f}{\bar{\mathbf{T}}^{ry}(1)} \cdot \mathbf{1}_n \end{array} \right] = y_f \cdot \mathbf{1}_n \quad (22)$$

where $\mathbf{T}^{ry} \in \mathbb{R}^{k_f \times k_f}$ is defined by $\mathbf{T}^{ry}{}_{ij} := h_{ry}(i-j)$, $\bar{\mathbf{T}}^{ry} \in \mathbb{R}^{n\times(k_f+n)}$ by $\bar{\mathbf{T}}^{ry}_{ij} := h_{ry}(i + k_f - j)$ and $\mathbf{T}^{ru} \in \mathbb{R}^{k_f\times k_f}$ by $\mathbf{T}^{ru}{}_{ij} := h_{ru}(i-j)$.

Proof.(Necessity) Assume that there exists a vector $\mathbf{r}$ for which equations (20)–(22) are satisfied. Define the command sequence

$$r(k) = \begin{cases} 0 \text{ if } k < 0 \\ \mathbf{r}_k \text{ if } 0 \leq k < k_f \\ \frac{y_f}{T_{ry}(1)} \text{ if } k \geq k_f \ . \end{cases} \quad (23)$$

The output is given by $y(k) = \sum_{i=0}^{\infty} r(k-i)h_{ry}(i)$. Setting $\mathbf{y} \in \mathbb{R}^{k_f} : \mathbf{y}_i = y(i)$ and $\bar{\mathbf{y}} \in \mathbb{R}^n : \bar{\mathbf{y}}_i = y(k_f + i)$, it is

$$\mathbf{y} = \mathbf{T}^{ry}\mathbf{r}\ ,\ \bar{\mathbf{y}} = \bar{\mathbf{T}}^{ry} \left[\begin{array}{c} \mathbf{r} \\ \hline \dfrac{y_f}{T_{ry}(1)} \cdot \mathbf{1}_q \end{array} \right] ,$$

and, by (20), $y(k)$ satisfies Property (17) of Definition 2, $\forall k < k_f$. It remains to show that $y(i) = y_f$, $\forall i \geq k_f$. To prove this, consider the reference-output pair $(r_1(k),\ y_1(k)) = (\frac{y_f}{T_{ry}(1)},\ y_f)$, $\forall k \in \mathbb{Z}$. Consider the reference $r_2(k) = r(k) - r_1(k)$, which is null if $k \geq k_f$. By linearity, the corresponding output is given by $y_2(k) = y(k) - 1$, with $y_2(\kappa + i) = 0$, $\forall i \in 0, \dots, n-1$, therefore at sample time k_f, the output that corresponds to the input u_2 is the solution of a degree n homogeneous difference equation with null initial conditions, therefore $y_2(k) = y(k) - 1 = 0$ is identically zero for $k \geq k_f$ and $y(k) = 1$, $\forall k \geq k_f$. Finally setting $\mathbf{u} \in \mathbb{R}^{k_f} : \mathbf{u}(k) = u(k)$ it is

$$\mathbf{u} = \mathbf{T}^{ru}\mathbf{r}\ ,$$

moreover by (21), property (18) is satisfied and $u(k) = \frac{y_f}{P(0)}$ if $k > k_f$.

(Sufficiency) Assume that for a given k_f, there exists a couple $(r(k), y(k)) \in \mathcal{K}_{\mathbf{p}}$. Define $\mathbf{r}$, $\mathbf{u}$ and $\mathbf{y}$ as above, by properties (15) and (17) it follows that

$$u_c^- \cdot \mathbf{1}_{k_f} < \mathbf{u} < u_c^+ \cdot \mathbf{1}_{k_f}$$
$$y_c^- \cdot \mathbf{1}_{k_f} < \mathbf{y} < y_c^+ \cdot \mathbf{1}_{k_f}\ ,$$

moreover, being $y(k) = \sum_{i=0}^{+\infty} h_{ry}(k-i)r(i)$,

$$\left[\begin{array}{c} \mathbf{y} \\ \hline \bar{\mathbf{y}} \end{array} \right] = \left[\begin{array}{c|c} \mathbf{T}^{ry} & 0 \\ \hline \multicolumn{2}{c}{\bar{\mathbf{T}}^{ry}} \end{array} \right] \left[\begin{array}{c} \mathbf{r} \\ \hline \dfrac{y_f}{T_{ty}(1)} \cdot \mathbf{1}_n \end{array} \right] ,$$

therefore equations (20)–(22) are satisfied. □

IV. SIMULATION RESULTS

The approach presented in Section III has been applied in simulation to the following two examples. In both cases an output transition from 0 to 1 is required and $u_c^- = 0$, $u_c^+ = 2$, $y_c^- = -0.05$, $y_c^+ = 1.05$ has been fixed.

A. Example 1

In the first example the following plant has been considered:

$$P(s) = \frac{1}{s+1}e^{-0.5s}. \quad (24)$$

The controller is an output-filtered PID controller with the value of the proportional gain K_p equal to 2, that of the integral time constant T_i equal to 1 and that of the derivative time constant T_d equal to 0.25. The filter time constant is selected as $T_f = 0.01$ such that the filter dynamics does not influence the control system dynamics. Thus, we have

$$C(s) = 2\left(1 + \frac{1}{s} + 0.25s\right)\frac{1}{0.01s+1}. \quad (25)$$

The application of the proposed methodology provides the closed-loop command input (solid line) and the process output (dashed line) shown in Figure 3 and the control variable shown in Figure 4. It can be seen that the minimum-time transition is performed in 4.15 s. It can be noted that the posed constraints are not exceeded and that the presence of the overshoot on the process output is due to the fact that a rest-to-rest transition is achieved. Furthermore the control

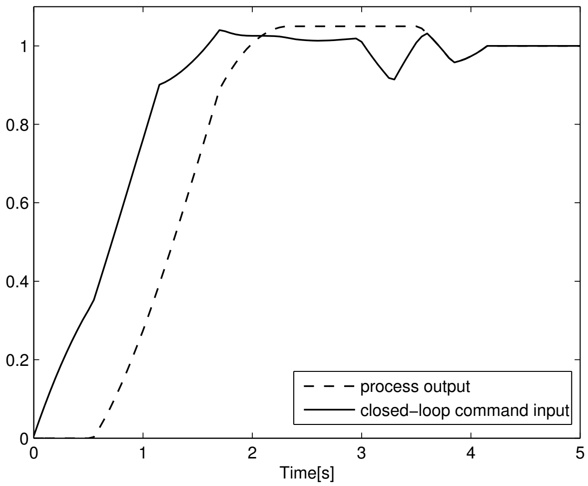

Fig. 3. Results for example 1.

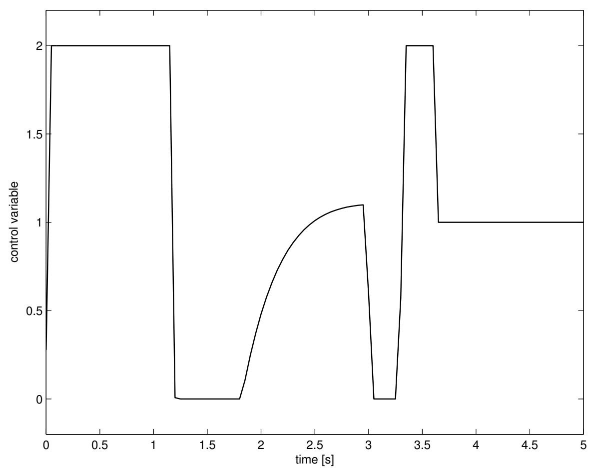

Fig. 4. Control variable for example 1.

variable is composed of intervals of input saturation (bang-bang control) and intervals of output saturation and input zero dynamics (see [15]).

B. Example 2

In the second example the (high-order) plant and the output-filtered PID controller are given by

$$P(s) = \frac{1}{(s+1)^4} e^{-0.5s}, \tag{26}$$

$$C(s) = 1.07 \left(1 + \frac{1}{4.76s} + 1.19s\right) \frac{1}{0.01s+1}. \tag{27}$$

The simulation results are presented in Figure 5 and 6. In this case the optimal transition time is 21.4 s. The same conclusions related to example 1 can be drawn also in this case, despite the different system dynamics. In this case the control variable always saturates, as in bang-bang control. It is worth stressing again that using the approach presented in this paper, both the controller and the plant reach the equilibrium in minimum time.

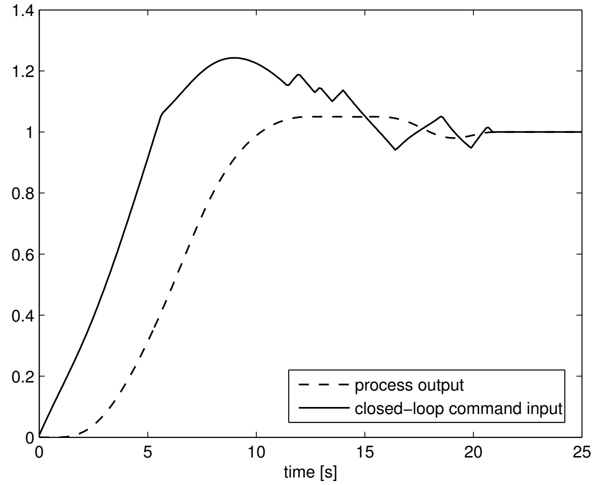

Fig. 5. Results for example 2.

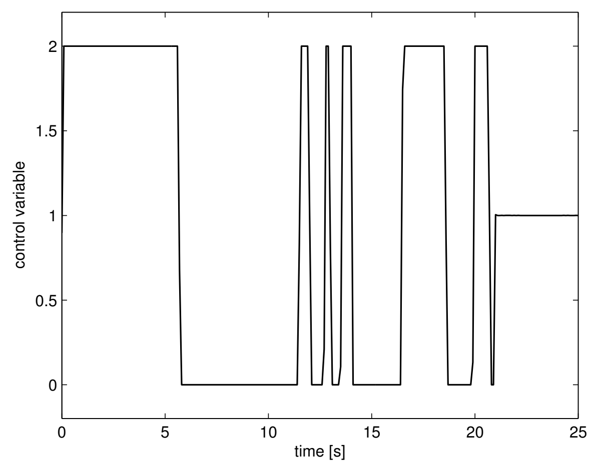

Fig. 6. Control variable for example 2.

V. EXPERIMENTAL RESULTS

In order to prove the effectiveness of the devised technique in practical applications, a laboratory experimental setup (made by KentRidge Instruments) has been employed (see Figure 7). Specifically, the apparatus consists of small perspex tower-type tank (whose area is 40 cm^2) in which a level control is implemented by means of a PC-based controller. The tank is filled with water by means of a pump whose speed is set by a DC voltage (the manipulated variable), in the range 0-5 V, through a PWM circuit. The tank is fitted with an outlet at the base in order for the water to return to a reservoir. The measure of the level of the water is given by a capacitive-type probe that provides an output signal between 0 (empty tank) and 5 V (full tank). Despite the model is nonlinear (since the output flow rate depends on the square root of the level), a FOPDT model has been estimated by applying the area method to the open-loop response with a step from 2 V to 2.5 V at the process input. The FOPDT

Fig. 7. The experimental setup for the level control experiment (only one tank has been adopted).

model obtained is

$$P(s) = \frac{1}{20s+1}e^{-2s}$$

Based on this model, it has been set $K_p = 7.41$, $T_i = 20$ and $T_d = 0$. The derivative action has not been employed due to the excessive measurement noise. A process variable transition from 2 V to 3 V (starting when the process is at the steady-state) has been required. After having fixed the minimum and maximum limit on the control variable to 0 V and 4.2 V respectively ($u_c^- = 0$, $u_c^+ = 4.2$) and a maximum overshoot of 5% ($y_c^- = 0$, $y_c^+ = 3.15$) , the command input has been determined by applying the algorithm described in Section III (see Figure 8). The resulting process variable and control variable are plotted in Figures 10 and 9 respectively. It appears that the control variable slightly exceed the imposed limit of 4.2 V but the process output transient is satisfactory (indeed the overshoot is almost negligible). For the sake of comparison, the response of the system to a step set-point signal is shown in Figure 11 (the corresponding control signal is shown in Figure 12). A larger overshoot appears.

VI. Conclusions

A linear programming approach to the common signal design for a PID control system has been presented. The approach can achieve a minimum-time rest-to-rest transition for regulated process output under arbitrary stringent constraints on the input and the output of the plant. The presented simulations and experimental results show the effectiveness of the overall methodology.

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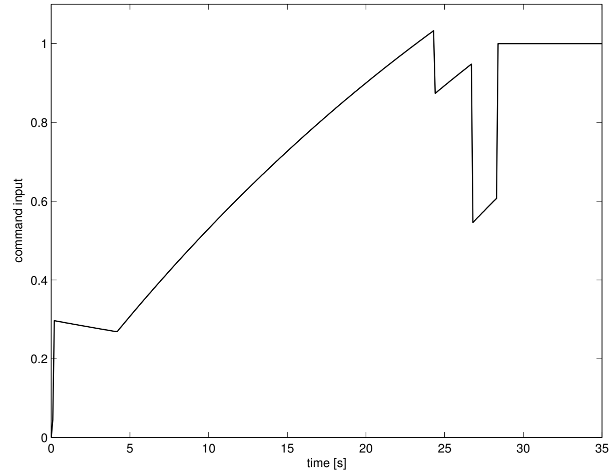

Fig. 8. The optimal command input for the level control experiment.

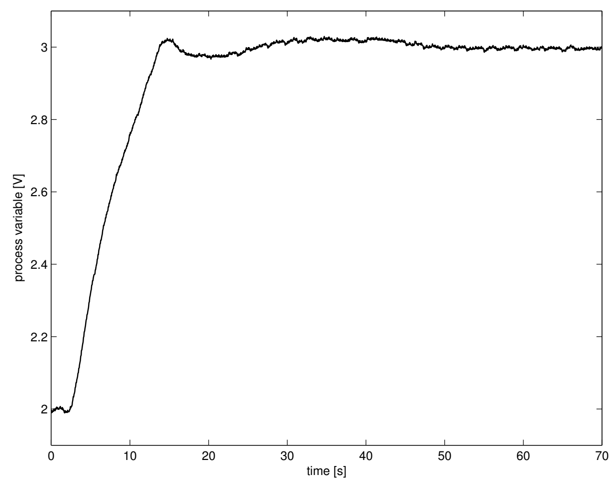

Fig. 9. The resulting process variable for the level control experiment.

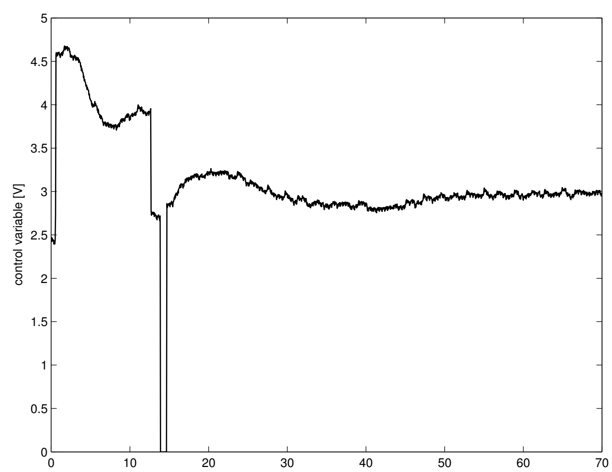

Fig. 10. The resulting control variable for the level control experiment.

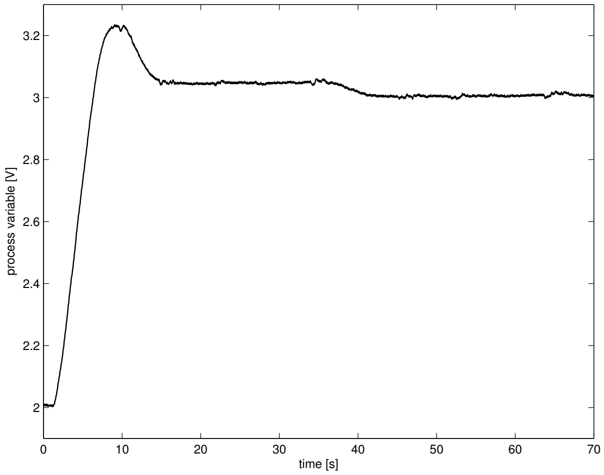

Fig. 11. The resulting process variable for the level control experiment with a step set-point signal.

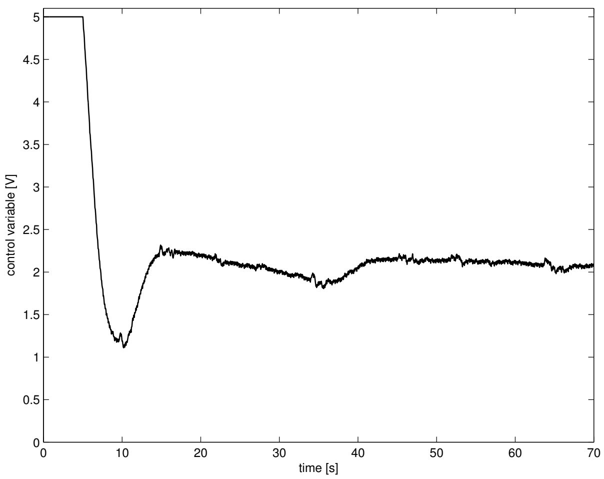

Fig. 12. The resulting control variable for the level control experiment with a step set-point signal.

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