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A noncausal approach for PID control ☆

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Abstract

A new approach for the improvement of the set-point following performances achieved by a PID controller is presented. Basically, it consists of applying a suitable command signal to the closed-loop control system in order to achieve a desired transient response when the process output is required to assume a new value. This command signal is determined by means of a stable input–output procedure for which a closed-form solution is presented, thus making the technique suitable to implement in the industrial context. Simulation and experimental results show that high performances are obtained despite the presence of model uncertainties and, above all, almost independently on the PID tuning. Thus, the PID gains can be selected in order to guarantee good load rejection performances without impairing the set-point transient responses.

Keywords: PID control; Stable input–output inversion; Feedforward control

1. Introduction

Proportional-Integral-Derivative (PID) controllers are undoubtedly the most adopted controllers in industry, due to the good cost/benefit ratio they are able to provide. To help the operator to select the controller gains to address given control specifications, many tuning formulas have been devised in the past [1] and autotuning functionalities are almost always available in commercial products [2,3]. However, it is well known that good performances both in the set-point following and in the load disturbances rejection task are often difficult to achieve at the same time. In order to solve this problem, which is of concern in many applications, the typical approach is to adopt a two degrees-of-freedom controller, namely, to adopt a feedforward (linear) compensator [4]. The use of the well-known set-point weighting strategy [5] falls in this framework. The main disadvantage of this method is that the reduction of the overshoot is paid by a slower set-point response. To overcome this drawback, the use of a variable set-point weight [6,7] or of a feedforward action [3,8–10] has been proposed.

From another point of view, it is well known that when the desired output trajectory is known in advance, the performance of a control system can be significantly improved by adopting a feedforward action determined by means of a stable inversion technique [11]. Note that the concept of dynamic input–output inversion [12,13] has been already proven to be effective in different areas of the automatic control field, such as motion control [14,15], flight control [16], robust control [17,18].

In this context we propose to use an analytic stable input–output inversion procedure to determine a suitable command signal to be applied to the closed-loop (PID based) control system, instead of the typical step signal, in order to achieve a high performance (i.e. low rise time and low overshoot at the same time) when the process

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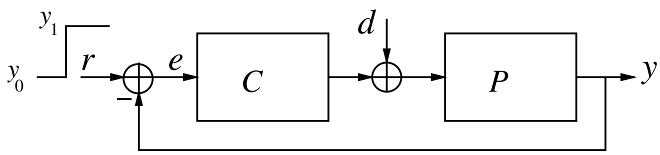

Fig. 1. The classic one degree-of-freedom PID based control scheme.

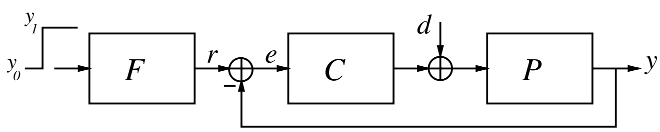

Fig. 2. The typical two degrees-of-freedom PID based control scheme.

output is required to assume a new value. To better understand the framework of the proposed method and the differences with the usual approaches, assume that the process variable is required to assume a steady-state value y_1 starting from a steady-state value y_0. The standard unity-feedback control scheme based on a one degree-of-freedom PID controller is shown in Fig. 1, where P is the process, y is the process output and C is the PID controller. The typical two degrees-of-freedom PID based control scheme is reported in Fig. 2, where F is a causal filter; note that the use of a set-point weight is equivalent to filtering the step signal to be applied to the closed-loop system with a second-order system [19]. The control scheme related to the new technique is shown in Fig. 3. Conversely to the other methods, here a step signal is not employed, but the knowledge in advance of y_1 is adopted by a command signal generator block to calculate, with a suitable preview time, a command signal r to be applied to the closed-loop PID control system in order to guarantee a high performance transient response.

Basically, the method consists of choosing a desired polynomial function to achieve a process output transition from y_0 to y_1 and then determining the command function r that causes the desired transition by inverting the closed-loop dynamics by means of a stable input–output inversion procedure.

The paper is organised as follows. The overall methodology is explained in Section 2 and discussed in Section 3. Illustrative results are shown in Section 4, while the robustness of the approach together with the role played by the unique design parameter are analysed in Section 5. Experimental results are presented in Section 6 and conclusions are drawn in the last section.

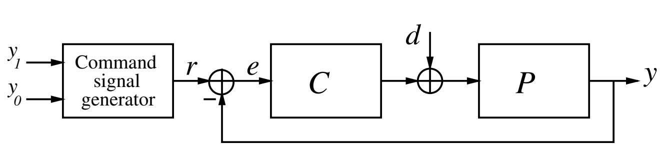

Fig. 3. The new dynamic inversion based control scheme.

2. Methodology

2.1. Modelling

As a first step of the devised method, the process to be controlled (assumed to be self-regulating) is modelled as a first-order plus dead-time (FOPDT) transfer function, i.e.

$$P(s;K,T,L) = \frac{K}{Ts+1}\,\mathrm{e}^{-Ls}. \tag{1}$$

This is a typical choice in industrial practice and a variety of methods, based on simple experiments, for the identification of the parameters K, T and L are available (see Section 5). Then, in order to have a rational transfer function, the dead-time term is approximated by means of a second-order Padè approximation. In this way, the approximated process transfer function results to be

$$\tilde{P}(s;K,T,L) = \frac{K}{Ts+1}\,\frac{1-Ls/6+L^2s^2/12}{1+Ls/6+L^2s^2/12}. \tag{2}$$

Note that if the process is nonself-regulating, it can be modelled as an integrator plus dead-time (IPDT) transfer function, i.e.

$$\tilde{P}(s;K,L) = \frac{K}{s}\,\frac{1-Ls/6+L^2s^2/12}{1+Ls/6+L^2s^2/12}. \tag{3}$$

Then, the methodology is basically the same for the FOPDT case and details for this case will be omitted in the following (an example will be presented in Section 4.3).

2.2. Tuning the PID controller

In order to apply the input–output inversion based methodology that will be presented in the following, the PID controller can be tuned according to any of the many methods proposed in the literature or even by a trial-and-error procedure. However, since the purpose of the proposed procedure is the attainment of high performances in the set-point following task, disregarding of the controller gains, it is sensible to select the PID parameters aiming only at obtaining good load rejection performances.

The PID controller transfer function be denoted as follows:

$$C(s;K_\mathrm{p},T_\mathrm{i},T_\mathrm{d},T_\mathrm{f}) = K_\mathrm{p}\left(1+\frac{1}{T_\mathrm{i}s}+T_\mathrm{d}s\right)\frac{1}{T_\mathrm{f}s+1} \tag{4}$$

where K_p is the proportional gain, T_i is the integral time constant, T_d is the derivative time constant and T_f is the time constant of a first-order filter that makes the transfer function proper.

2.3. Output function design

As a desired output function that defines the transition from a set-point value y_0 to another y_1 (to be performed in the time interval $[0,\tau]$) we choose a "transition" polynomial [21], i.e. a polynomial function that satisfies boundary conditions and that is parameterised by the transition time τ. In the following, without loss of generality we will assume $y_0 = 0$, Formally, define

$$y_d(t) = c_{2k+1}t^{2k+1} + c_{2pk}t^{2k} + \cdots + c_1 t + c_0$$

The polynomial coefficients can be uniquely found by solving the following linear system, in which boundary conditions at the endpoints of interval $[0, \tau]$ are imposed:

$$\begin{cases} y_d(0) = 0; & y_d(\tau) = y_1 \\ y_d^{(1)}(0) = 0; & y_d^{(1)}(\tau) = 0 \\ \vdots & \\ y_d^{(k)}(0) = 0; & y_d^{(k)}(\tau) = 0 \end{cases}$$

The results can be expressed in closed-form as follows ($t \in [0, \tau]$):

$$y_d(t;\tau) = y_1 \frac{(2k+1)!}{k!} \times \sum_{i=k+1}^{2k+1} \frac{(-1)^{i-k-1}}{(i-k-1)!(2k+1-i)!i} \left(\frac{t}{\tau}\right)^i \tag{5}$$

Expression (5) represents a monotonic function with neither undershooting nor overshooting and its use is therefore very appealing in the context of process control.

The order of the polynomial can be selected by imposing the order of continuity of the command input that results from the dynamic inversion procedure [21]. Specifically, if the plant is modelled as a FOPTD transfer function (see (1)), its relative degree is equal to one. Taking into account that the relative degree of the PID controller is zero, the relative degree of the overall closed-loop system is one. Thus, a third order polynomial ($k = 1$) suffices if a continuous command input function is required, i.e.

$$y_d(t;\tau) = y_1\left(-\frac{2}{\tau^3}t^3 + \frac{3}{\tau^2}t^2\right). \tag{6}$$

Outside the interval $[0, \tau]$ the function $y(t;\tau)$ is equal to 0 for $t < 0$ and equal to y_1 for $t > \tau$.

2.4. Stable input–output inversion algorithm

At this point we address the problem of finding the command signal $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ that provides the desired output function (6). For the sake of clarity of notation, in the following we will omit the dependence of the functions and of the resulting coefficients from the parameters K, T, L, K_p, T_i, T_d, T_f. The closed-loop transfer function be denoted as

$$H(s) := \frac{C(s)\tilde{P}(s)}{1 + C(s)\tilde{P}(s)} = K_1 \frac{b(s)}{a(s)} \tag{7}$$

where $b(s)$ and $a(s)$ are monic polynomials. As $H(s)$ is nonminimum phase, a stable dynamic inversion procedure is necessary, that is a bounded input function has to be found in order to produce the desired output [20]. Denote by $\mathscr{B}$ the behaviour set [22] of the closed-loop system:

$$\begin{aligned} \mathscr{B} := \{ & (r(\cdot), y(\cdot)) \in \mathrm{PC}(\mathbb{R}) \times \mathrm{PC}(\mathbb{R}) \\ & : y(\cdot) \text{ is the response output to input } u(\cdot) \\ & \text{for system } H(s)\} \end{aligned}$$

where $\mathrm{PC}(\mathbb{R})$ denotes the set of real piecewise-continuous functions defined over $\mathbb{R} = (-\infty, +\infty)$. Now, in order to perform the stable inversion, we rewrite the numerator of the transfer function (7) as follows:

$$b(s) = b_-(s)b_+(s)$$

where $b_-(s)$ and $b_+(s)$ denote the polynomials associated to the zeros with negative real part (i.e. those of the PID controller) and positive real part (i.e. those of the Padè approximation) respectively. From (2) we have

$$b_+(s) = (s - z_R^+)^2 + z_I^{+^2} \tag{8}$$

where $Z_R^+ = 3/L, Z_I^+ = \sqrt{3}/L$ correspond to the complex zeros $z_R^+ \pm jz_I^+ \in \mathbb{C}_+$. From (4) we distinguish three cases (depending on the selected PID parameters):

$$b_-(s) = (s - z_1^-)(s - z_2^-) \tag{9}$$

$$b_-(s) = (s - z^-)^2 \tag{10}$$

$$b_-(s) = (s - z_R^-)^2 + z_I^{-^2} \tag{11}$$

corresponding to real distinct zeros (9), real coincident zeros (10), and complex zeros (11), respectively. Now, consider the inverse system of (7) whose transfer function can be written as

$$H(s)^{-1} = \gamma_0 + \gamma_1 s + H_0(s) \tag{12}$$

where γ_0 and γ_1 are suitable constants and $H_0(s)$, a strictly proper rational function, represents the zero dynamics. This can be uniquely decomposed according to

$$H_0(s) = H_0^-(s) + H_0^+(s) = \frac{c(s)}{b_-(s)} + \frac{d(s)}{b_+(s)} \tag{13}$$

where $c(s) = c_1 s + c_0$ and $d(s) = d_1 s + d_0$ are first-order polynomials with coefficients depending on K, T, L, K_p, T_i, T_d and T_f. The modes associated to $b^-(s)$ and $b^+(s)$ be denoted by $m_i^-(t)$, $i = 1, 2$, and by $m_i^+(t)$, $i = 1, 2$, respectively. More specifically, the unstable zero modes are given by

$$m_1^+(t) = \mathrm{e}^{z_R^+ t} \cos z_I^+ t \quad m_2^+(t) = \mathrm{e}^{z_R^+ t} \sin z_I^+ t \tag{14}$$

while the stable zero ones are given according to the cases (9)–(11) by

$$m_1^-(t) = \mathrm{e}^{z_1^- t}, \quad m_2^-(t) = \mathrm{e}^{z_2^- t} \tag{15}$$

$$m_1^-(t) = \mathrm{e}^{z^- t}, \quad m_2^-(t) = t\mathrm{e}^{z^- t} \tag{16}$$

$$m_1^-(t) = \mathrm{e}^{z_R^- t} \cos z_I^- t, \quad m_2^-(t) = \mathrm{e}^{z_R^- t} \sin z_I^- t \tag{17}$$

Being $\mathscr{L}$ the Laplace transform operator, define:

$$\eta_0^-(t) := \mathscr{L}^{-1}[H_0^-(s)], \quad \eta_0^+(t) := \mathscr{L}^{-1}[H_0^+(s)].$$

The following propositions and the following theorem represent the solution to the stable dynamic inversion problem.

Proposition 1

$$\int_0^t \eta_0^+(t-v)y_d(v;\tau)\,\mathrm{d}v = H_0^+(0)y_d(t;\tau) + \frac{1}{\tau^3}(p_1^+(\tau)m_1^+(t)+p_2^+(\tau)m_2^+(t)) + \frac{1}{\tau^3}T_0^+(t;\tau) \quad (18)$$

where

$$T_0^+(t,\tau)=\begin{cases} s_0^+(t)+s_1^+(t)\tau & \text{if } t\in[0,\tau] \\ q_1^+(\tau)m_1^+(t-\tau)+q_2^+(\tau)m_2^+(t-\tau) & \text{if } t>\tau \end{cases} \quad (19)$$

and $p_i^+(\tau), q_i^+(\tau),\ i=1,2$ *are suitable* τ*-polynomials and* $s_i^+(t),\ i=0,1$ *are suitable t-polynomials.*

Proposition 2

$$\int_0^t \eta_0^-(t-v)y_d(v;\tau)\,\mathrm{d}v = H_0^-(0)y_d(t;\tau) + \frac{1}{\tau^3}(p_1^-(\tau)m_1^-(t)-p_2^-(\tau)m_2^-(t)) + \frac{1}{\tau^3}T_0^-(t;\tau) \quad (20)$$

where

$$T_0^-(t,\tau)=\begin{cases} s_0^-(t)+s_1^-(t)\tau & \text{if } t\in[0,\tau] \\ q_1^-(\tau)m_1^-(t-\tau)+q_2^-(\tau)m_2^-(t-\tau) & \text{if } t>\tau \end{cases} \quad (21)$$

and $p_i^-(\tau), q_i^-(\tau),\ i=1,2$ *are suitable* τ*-polynomials and* $s_i^-(t),\ i=0,1$ *are suitable t-polynomials.*

Theorem 1. *The function* $r(t;\tau)$ *defined as*

$$r(t;\tau) = -\frac{1}{\tau^3}(p_1^+(\tau)m_1^+(t)+p_2^+(\tau)m_2^+(t) - q_1^+(\tau)m_1^+(t-\tau)-q_2^+(\tau)m_2^+(t-\tau)) \quad \text{if } t<0 \quad (22)$$

$$r(t;\tau) = \gamma_1\dot{y}_d(t;\tau)+\gamma_0 y_d(t;\tau)+H_0(0)y_d(t;\tau) + \frac{1}{\tau^3}(s_0^+(t)+s_0^-(t)+s_1^+(t)\tau+s_1^-(t)\tau - q_1^+(\tau)m_1^+(t-\tau)-q_2^+(\tau)m_2^+(t-\tau)+p_1^-(\tau)m_1^-(t) + p_2^-(\tau)m_2^-(t)) \quad \text{if } t\in[0,\tau] \quad (23)$$

$$r(t;\tau) = \gamma_0+H_0(0)+\frac{1}{\tau^3}(p_1^-(\tau)m_1^-(t)+p_2^-(\tau)m_2^-(t) + q_1^-(\tau)m_1^-(t-\tau)+q_2^-(\tau)m_2^-(t-\tau)) \quad \text{if } t>\tau \quad (24)$$

is bounded over $(-\infty,+\infty)$ *and* $(r(t;\tau), y_d(t;\tau))\in\mathscr{B}$.

Proofs of the above propositions and of the above theorem can be derived straightforwardly from proofs reported in [20]. Explicit closed-form expressions of the polynomials involved in (22)–(24) are reported in the Appendix.

Summarizing, the determined function $r(t; K, T, L, K_\mathrm{p}, T_\mathrm{i}, T_\mathrm{d}, T_\mathrm{f}, \tau)$ exactly solves the stable inversion problem for FOPDT plants (in which the dead-time term has been approximated) controlled by a PID controller (4) and for a family of output functions, which depend on the free transition time τ.

Actually, from a practical point of view, since the synthesised function (22)–(24) is defined over the interval $(-\infty,+\infty)$, it is necessary to adopt a truncated function $r_a(t;\tau)$, resulting therefore in an approximate generation of the desired output $y_d(t;\tau)$. In particular, a preactuation time t_s and a postactuation time t_f can be selected so that $r_a(t;\tau)=0$ for $t<t_s$ and $r_a(t;\tau)=y_1$ for $t>t_p$. By taking into account that the preactuation and postactuation inputs (i.e. the input defined for $t<0$ and $t>\tau$, respectively) converge exponentially to zero at time $t\to-\infty$ and to y_1 at time $t\to+\infty$, an arbitrarily precise approximation can be accomplished [20]. Practically, the method suggested in [23] can be adopted. It consists of selecting

$$t_s = -\frac{10}{D_{\mathrm{rhp}}}, \quad t_p = \frac{10}{D_{\mathrm{lhp}}} \quad (25)$$

where D_{rhp} and D_{lhp} are the minimum distance of the right and left half plane poles, respectively from the imaginary axis of the complex plane.

Hence, the approximate command signal to be actually used is

$$r_a(t;\tau) := \begin{cases} 0 & \text{for } t<t_s \\ r(t;\tau) & \text{for } t_s\leqslant t\leqslant t_p \\ y_1 & \text{for } t>t_p. \end{cases}$$

Remark 1. It is worth highlighting that the preactuation time depends only on the (apparent) dead time of the process, as this determines the unstable zeros of the closed-loop systems by means of the Padè approximation. Conversely, the postactuation time depends on the tuning of the PID parameters because the stable zeros of the closed-loop systems are those of the controller.

Remark 2. The preactuation time corresponds to a preview time $|t_s|$ that is the anticipation time necessary to command the desired set-point transition on the controlled output.

3. Discussion

The overall stable input–output inversion procedure presented in Section 2.4 can be performed by means of a symbolic computation, i.e. a closed-form expression of $r(t; K, T, L, K_\mathrm{p}, T_\mathrm{i}, T_\mathrm{d}, T_\mathrm{f}, \tau)$ results. Indeed, the actual command signal to be applied for a given plant and a given controller is determined by substituting the actual value of the parameters into the resulting closed-form expression.

In this context, the choice of using a second-order Padè approximation is motivated, from one side, by keeping the expression of $r(t; K, T, L, K_\mathrm{p}, T_\mathrm{i}, T_\mathrm{d}, T_\mathrm{f}, \tau)$ as simple as possible and, from the other side, by providing an approximation as good as possible, since the basic rationale of this method is to apply model-based feedforward control action.

In any case, it is worth noting that the presented inversion procedure is based on a general one [20], where $H(s)$ can be the rational transfer function of any (stable) system, provided that there are not purely imaginary zeros. Thus, as already mentioned, the proposed approach can be straightforwardly applied also to integral (and unstable) processes $\tilde{P}(s)$, as it is based on the inversion of the dynamics of the closed-loop system $H(s)$ (see Sections 4.3 and 4.4). Analogously, the same method can be trivially extended to PI, P and PD control. Indeed, it appears that the devised method can be extended also to high-order processes. Thus, a more accurate model of the process, if available, can be fully exploited. However, in this case, the inversion procedure has to be performed on purpose and this might prevent the use of the method in single-station controllers. Conversely, if a FOPDT (or a IPDT) model is employed, the determined general closed-form expression of $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ can be used.

Once the PID controller has been tuned, the only free design parameter is the transition time τ. This value can be selected by solving an optimisation problem where the transition time has to be minimised subject to actuator constraints [21]. Alternatively, from a more practical point of view, the choice can be made by the user either directly or through a (possibly) more intuitive reasoning. For example, the user might select a ratio between the bandwidth of the open-loop system and that of the closed-loop one, from which the value of τ can be determined easily. In any case, parameter τ represent a very desirable feature from a user point of view, as it allows to handle the trade-off between performance, robustness and control activity [24,25]. This aspect will be analysed further in Section 5.

Finally, it is worth stressing that, differently from the set-point weighting approach (i.e. differently from filtering the set-point with a causal filter), the devised approach allows to recover the set-point following performances even in the case of a sluggish tuning of the PID controller (see Section 4).

4. Illustrative results

In the following examples the process output has to perform a transition from 0 to $y_1 = 1$.

4.1. FOPDT process

Consider the following FOPDT process:

$$P_1(s) = \frac{1}{10s+1} e^{-6s}. \tag{26}$$

To prove the effectiveness of the method with different PID tunings, three sets of PID parameters have been considered, namely, the one given by the Ziegler–Nichols step response PID formula ($K_p = 2$, $T_i = 12$, $T_d = 3$), the one given by the Ziegler–Nichols step response PI formula ($K_p = 1.5$, $T_i = 18$, $T_d = 0$), and the one that results from the minimisation of the ISTE integral criterion for the load disturbance rejection [26] ($K_p = 2.41$, $T_i = 7.33$, $T_d = 2.74$). In the first and in the third case it has been set $T_f = 0.01$ (for a PI controller the filter is not necessary) and the transition time has been always fixed to $\tau = 10$.

The resulting value of the preaction and postaction times in the three cases are $t_s = -20$ and $t_p = 60$, $t_p = 180$, $t_p = 54.8$, respectively (note that, for convenience, the time axis has been properly shifted in order to have $t_s = 0$). The determined command functions are reported in Fig. 4 (solid line: Ziegler–Nichols PID; dash-dot line: Ziegler–Nichols PI; dashed line: integral criterion minimisation) and the corresponding process outputs are plotted in Fig. 5. For the sake of comparison, the process outputs resulting with the classic method, i.e. by applying a step set-point signal, are also reported.

It appears that the inversion based methodology is able to provide low rise times and low overshoots at the same time but, most of all, is able to provide basically the same response despite a very different PI(D) tuning, as it is evidenced by the very different step responses they provide. Note that this is achieved with a lower control effort with respect to the classic case since the step signal is substituted by a smoother signal (plots of the control signals are not reported for the sake of brevity).

4.2. High-order process

As a second example the following high-order process is considered:

$$P_2(s) = \frac{1}{(s+1)^8}. \tag{27}$$

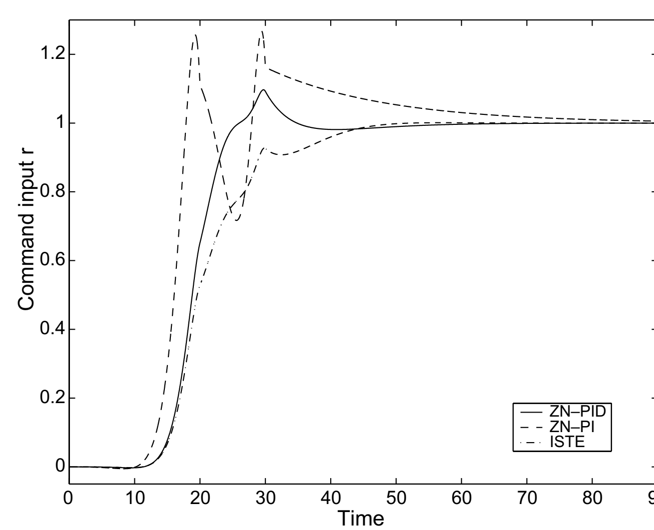

Fig. 4. The determined command signals for the three considered PID tuning for the FOPDT system.

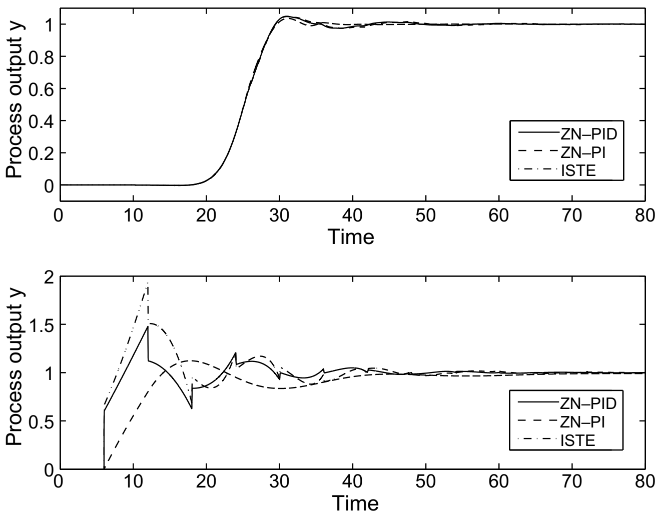

Fig. 5. The resulting process outputs with the new method (top) and with the classic method (bottom) for the three considered PID tuning for the FOPDT system.

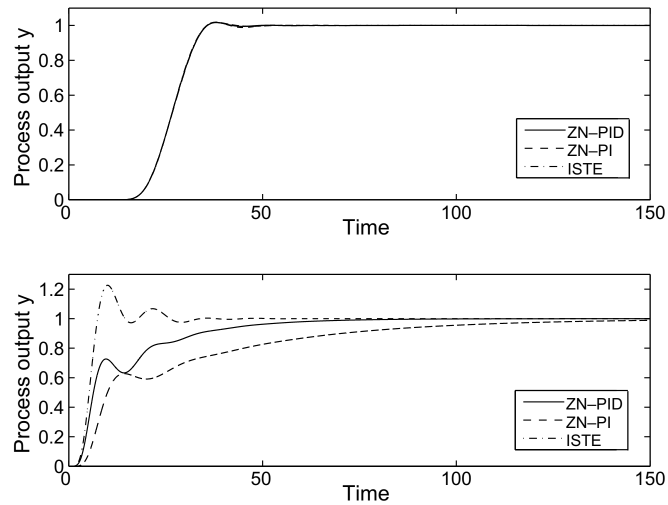

Fig. 7. The resulting process outputs with the new method (top) and with the classic method (bottom) for the three considered PID tuning for the high-order system.

By applying the area method [3], a FOPDT transfer function has been estimated, resulting in $K=1$, $T=3.04$ and $L=4.97$. With respect to these parameters, the same tuning formulae as for the FOPDT example has been adopted, resulting in $K_p=0.73$, $T_i=9.93$, $T_d=2.48$ ($T_f=0.01$) for the Ziegler–Nichols PID tuning, $K_p=0.55$, $T_i=14.90$, $T_d=0$ for the Ziegler–Nichols PI tuning, and $K_p=1.06$, $T_i=4.26$, $T_d=2.48$ ($T_f=0.01$) for the minimisation of the ISTE integral criterion. The transition time has been fixed to $\tau=20$ in all the cases.

The resulting values of the preaction and postaction times in the three cases are $t_s=-16.57$ and $t_p=51.23$, $t_p=149$, $t_p=49.6$, respectively. The command functions determined by applying the input–output inversion procedure are reported in Fig. 6 and the corresponding process outputs are plotted in Fig. 7, together with the process outputs resulting with the classic method, i.e. by applying a step set-point signal. The same line types as before have been adopted also in this example. The same considerations of the previous example can be done also on this case. Further, the robustness of the method with respect to model uncertainties appears. This aspect will be further analysed in Section 5.

4.3. IPDT process

As an example for IPDT processes, the following process has been considered [27]:

$$P_3(s)=\frac{0.0506}{s}e^{-6s}. \tag{28}$$

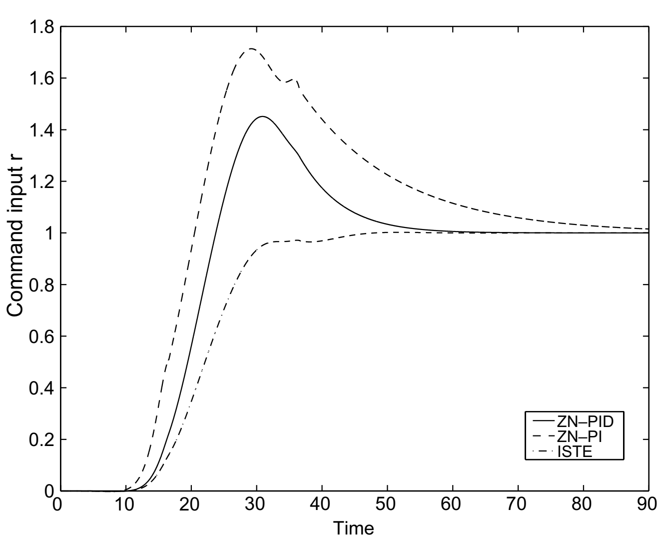

Fig. 6. The determined command signals for the three considered PID tuning for the high-order system.

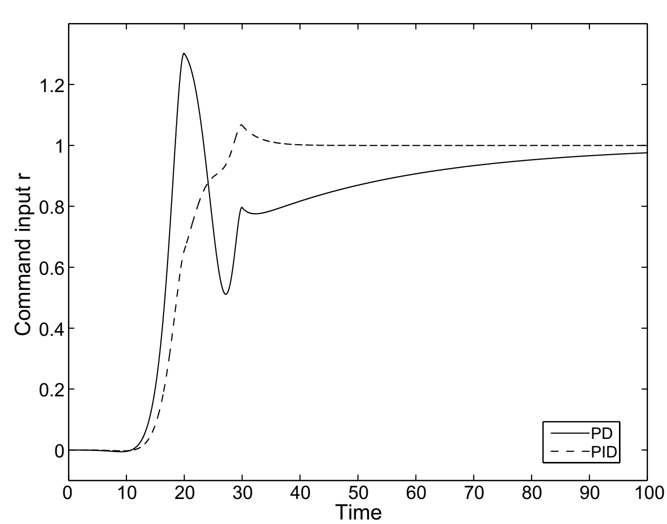

Fig. 8. The determined command signals for the PD and PID tuning for the IPDT system.

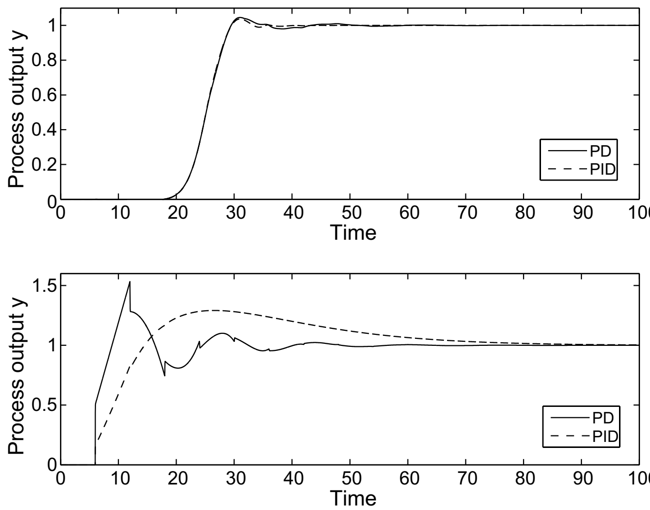

Fig. 9. The resulting process outputs with the new method (top) and with the classic method (bottom) for the PD and PID tuning for the IPDT system.

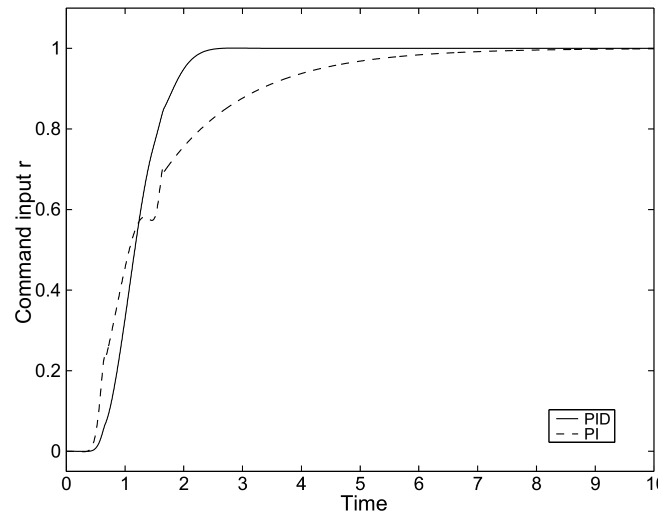

Fig. 10. The determined command signals for the PID and PI tuning for the unstable system.

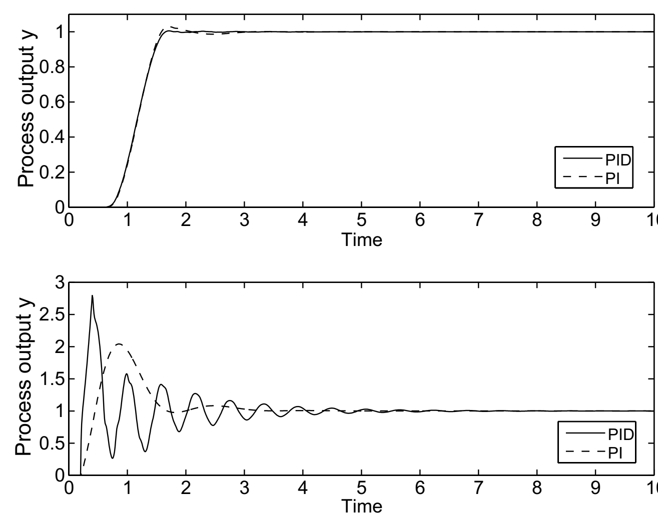

Fig. 11. The resulting process outputs with the new method (top) and with the classic method (bottom) for the PID and PI tuning for the unstable system.

Two sets of PID parameters have been adopted, namely, the one based on the minimisation of the integrated square error for set-point responses [28] (which is actually a PD controller with $K_\mathrm{p} = 3.394$ and $T_\mathrm{d} = 2.94$) and the one proposed in [27] ($K_\mathrm{p} = 2.0123$, $T_\mathrm{i} = 31.2030$, $T_\mathrm{d} = 1.5674$). In both cases it has been set again $T_\mathrm{f} = 0.01$ and $\tau = 10$.

The resulting value of the preaction and postaction times are $t_s = -20.01$ s and $t_p = 29.41$ and $t_p = 295.48$ for the PD and PID case, respectively. The determined command functions are reported in Fig. 8 and the corresponding process outputs are plotted in Fig. 9, together with the process outputs resulting from the application of a step set-point signal.

It appears that the same considerations that have been done for self-regulating processes can be done also for integrating processes. Indeed, almost the same performances are obtained despite the different tuning strategies that have been employed.

4.4. Unstable process

The devised technique can be straightforwardly applied also to unstable processes. Consider for example the process model by the following transfer function:

$$P_4(s) = \frac{1}{s-1}\mathrm{e}^{-0.2s}. \tag{29}$$

Also in this case, two sets of PID parameters have been selected, the one that results from the minimisation of the integrated square error of the load disturbance response [28] ($K_\mathrm{p} = 6.85$, $T_\mathrm{i} = 0.36$, $T_\mathrm{d} = 0.12$, $T_\mathrm{f} = 0.01$) and the one proposed in [29], i.e. $K_p = 3.46$ and $T_\mathrm{i} = 1.47$, which is actually a PI controller. The desired transition time has been fixed to $\tau = 1$.

Results are shown in Figs. 10 and 11. It can be seen that the devised methodology retains its effectiveness also for unstable processes and therefore its generality appears.

5. Practical issues

Since the proposed approach is based on the determination of a suitable model-based feed-forward action, it is necessary to evaluate the robustness of the approach with respect to the method adopted for the estimation of a FOPDT model influences the results. In addition, the role played by the unique design parameter τ has to be clarified. In this context, a remarkable theoretical result is that for $\tau \to +\infty$ the closed-loop system output tends to the inverse-based input, independently of the closed-loop system transfer function [20]. In any case, to better understand these practical issues, a few methods for the estimation of a FOPDT transfer function have been selected among the many methods that have been proposed and proven to be effective in the literature. They have been chosen based on the fact that they are suitable for industrial (PID) control, as they are based on simple and cost-effective experiments that are typically employed in industrial practice, namely, the application of a step signal to the process input (open-loop experiment) or of a relay feedback (closed-loop experiment). Note that, since the aim of this investigation is to evaluate the application of the different methods in the noncausal approach rather than evaluating and comparing the effectiveness of each method in estimate an accurate FOPDT model, measurement noise is not taken into account in the following.

5.1. Methods based on step response

The following methods based on the evaluation of an open-loop step response, where different approaches are exploited, have been considered:

(a) the well-known area method [3, p. 24];
(b) the least squares method proposed in [30], which is based on the integrated process input and output signals;
(c) the least squares method based on the use of Laguerre functions described in [31, chapter 2];
(d) the least squares method proposed in [32] applied to FOPDT transfer function.

It has to be noted that whereas methods (a) and (d) yield directly to a FOPDT transfer function, methods (b) and (c) actually yield to a rational transfer function of arbitrarily chosen order. Thus, in these cases, a fourth-order transfer function with relative order equal to one has been estimated first. Then, it has been reduced to a FOPDT model by adopting the model reduction method proposed in [30].

5.2. Methods based on relay feedback

Relay feedback based methods [33] are the most adopted methods where a closed-loop experiment is used for automatic tuning of PID controllers. Actually, the original idea is to employ the relay feedback to estimate the ultimate gain and the ultimate period of the system. However, recently, various techniques have been devised in order to estimate a FOPDT transfer function. The following ones have been considered in this paper:

(e) the standard method with a symmetric relay without hysteresis. The FOPDT transfer function is then determined by straightforward calculations (see for example [34, chapter 2]);
(f) the technique based on the use of a biased relay (with hysteresis) [33];
(g) the method described in [35] where two points on the Nyquist curve (the one where the phase is $-90°$ in addition to that where the phase is $-180°$) are estimated. The FOPDT transfer function parameters are then estimated by assuming the knowledge of the process gain (note that this can be simply estimated by considering steady-state values of the input and the output);
(h) the method in which an asymmetrical relay is adopted [36];
(i) the method based on the so-called curvature factor of the process response described in [37].

It turns out that the addressed techniques represent rather different approaches.

5.3. Illustrative results

In order to evaluate the influence of the estimation method in the proposed noncausal approach, consider, as an example, the following system:

$$P_5(s) = \frac{1}{(s+1)^4} \tag{30}$$

A FOPDT model has been estimated with all the considered techniques. Then, the PID controller has been tuned, by means of a genetic algorithm [38] in which the nominal process transfer function has been considered, in order to minimise the integrated absolute error defined as

$$IAE = \int |e(t)|\,\mathrm{d}t \tag{31}$$

where $e(t)$ is the difference between the set-point value and the process output, when a step load disturbance occurs on the control system. It results $K_\mathrm{p} = 3.50$, $T_\mathrm{i} = 1.77$, and $T_\mathrm{d} = 1.47$ ($T_\mathrm{f} = 0.01$). Thus, in order to analyse the effect of the identification method just on the inversion approach and not on the PID controller tuning, the best PID parameters (in the sense of those that provide the best load disturbance response) have been adopted. Hence, the tuning of the PID controller depends only on the considered process and it is independent from the adopted identification method (i.e. we have the same tuning disregarding the adopted estimation method).

Then, for each estimated process transfer function the command signal $r(t; K, T, L, K_\mathrm{p}, T_\mathrm{i}, T_\mathrm{d}, T_\mathrm{f}, \tau)$ has been calculated with different values of τ and two values have been calculated with respect to the inversion-based closed-loop response, namely, the integrated absolute error with respect to the desired final output value y_1 and the integrated absolute error with respect to the desired output response (6), i.e.

$$\mathrm{IAE}_a = \int_{t_0}^{+\infty} |y_1 - y(t)|\,\mathrm{d}t \tag{32}$$

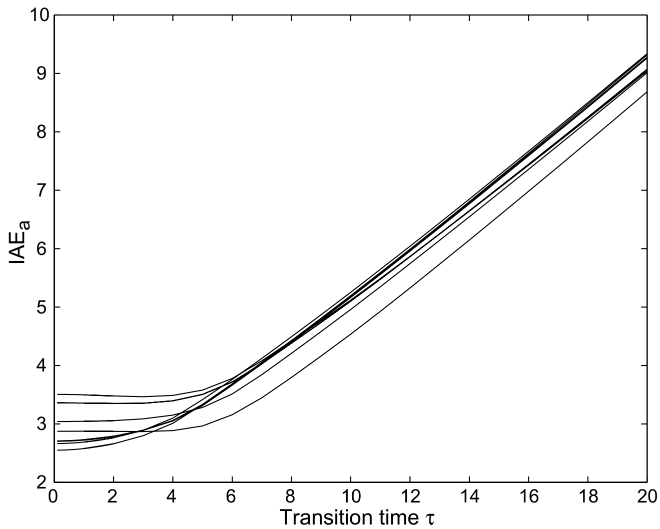

Fig. 12. IAE_a for different values of τ obtained by considering the nine different identification methods.

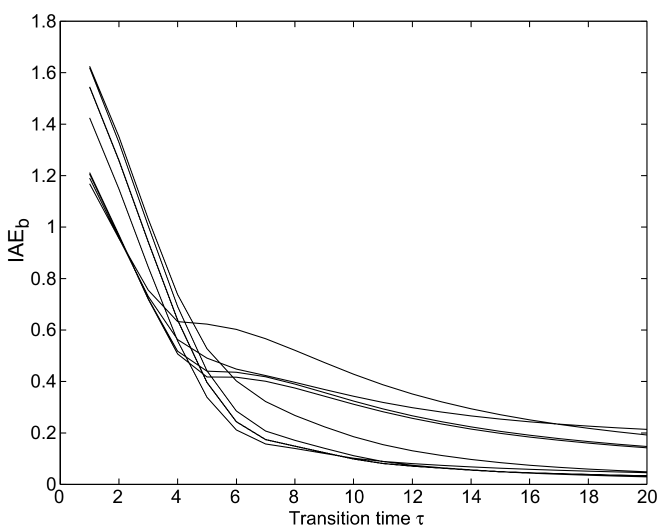

Fig. 13. IAE_b for different values of τ obtained by considering the nine different identification methods.

and

$$IAE_b = \int_{t_0}^{+\infty} |y_d(t) - y(t)|\,dt \tag{33}$$

where t_0 is the time instant in which the process output y leaves the zero value, so that the apparent dead time of the process and the preaction time do not bias the result.

The values of IAE_a and IAE_b for different values of the specified output transition time τ that have been obtained by employing the nine considered identification methods are plotted in Figs. 12 and 13, respectively. For clarity, the nine plots in each figure have not been labelled, because they are actually very difficult to distinguish in some cases. Indeed, it is worth stressing the general result more than providing detailed quantitative results. Note however that the value of IAE_a obtained by applying a step set-point signal instead of the inversion based command signal is 2.32. For a better analysis of the technique, Fig. 14 shows the different outputs obtained for increasing values of τ (i.e. $\tau = 1, 2, \ldots, 10$) when the process $P_5(s)$ has been estimated with method (a) (see Section 5.1). The corresponding control variable signals are plotted in Fig. 15.

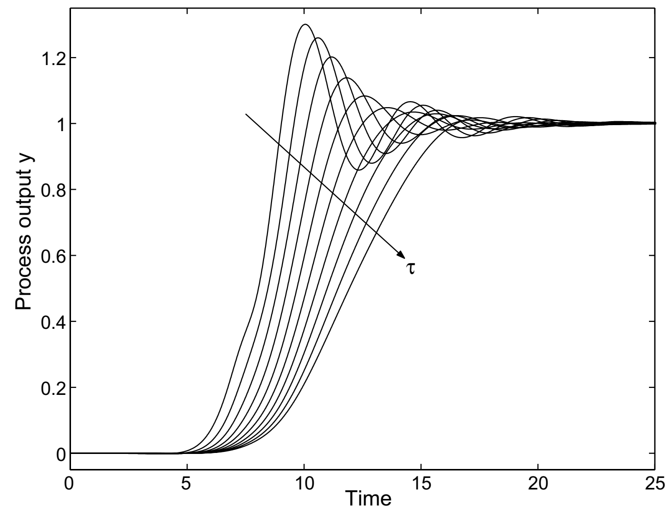

Fig. 14. Process $P_5(s)$ outputs with different values of τ.

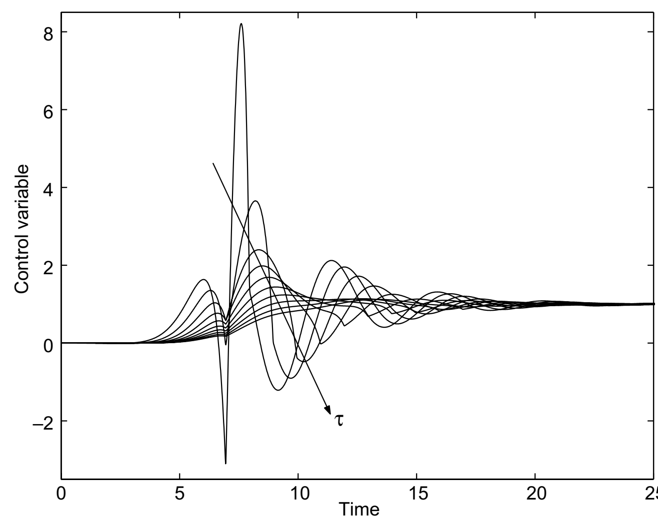

Fig. 15. Resulting control variable signal with different values of τ.

By taking into account that results obtained with other processes [39] are similar, it can be deduced that, as expected, the desired transition time τ handles the trade-off between robustness and aggressiveness (and control activity), independently from the adopted estimation method. Indeed, by increasing the value of τ the desired response is obtained more accurately (IAE_b decreases), although this implies that IAE_a increases as the rise time increases. In any case good performances are obtained for relatively small values of τ (thus, excessively high values of τ are not worth to being employed). Actually, if the PID controller is well tuned from the point of view of minimising the value of IAE_a for a step set-point signal, then the inversion based approach might give a higher value of IAE_a than using a step set-point signal. However, it has to be taken into account that a low value of the integrated absolute error is often achieved at the expense of high overshoots, high control activity and poor robustness and these latter aspects are almost always of major concern in the industrial context. Indeed, they can be handled by the devised noncausal approach. Finally, it cannot be said that one of the estimation methods is better than the others from the point of view of the noncausal approach. Actually, they provide basically the same performances and in case a method is worse than another for a given value of τ, it can be better for another value of the transition time.

6. Experimental results

In order to prove the effectiveness of the devised technique in practical applications, a laboratory experimental setup (made by KentRidge Instruments) has been employed (see Fig. 16). Specifically, the apparatus consists of small perspex tower-type tank (whose area is 40 cm^2) in which a level control is implemented by means of a PC-based controller. The tank is filled with water by means of a pump whose speed is set by a DC voltage (the manipulated variable), in the range 0–5 V, through a PWM circuit. The tank is fitted with an outlet at the base in order for the water to return to a reservoir. The measure of the level of the water is given by a capacitive-type probe that provides an output signal between 0 (empty tank) and 5 V (full tank). Since the apparent dead-time of the system

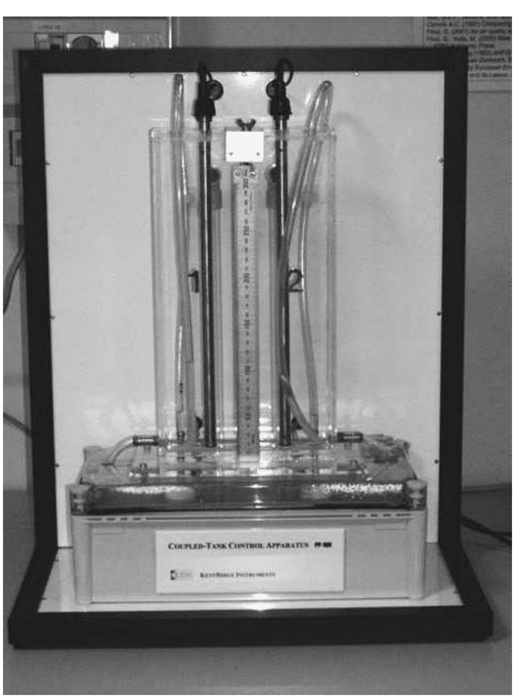

Fig. 16. The experimental setup for the level control experiment (only one tank has been adopted).

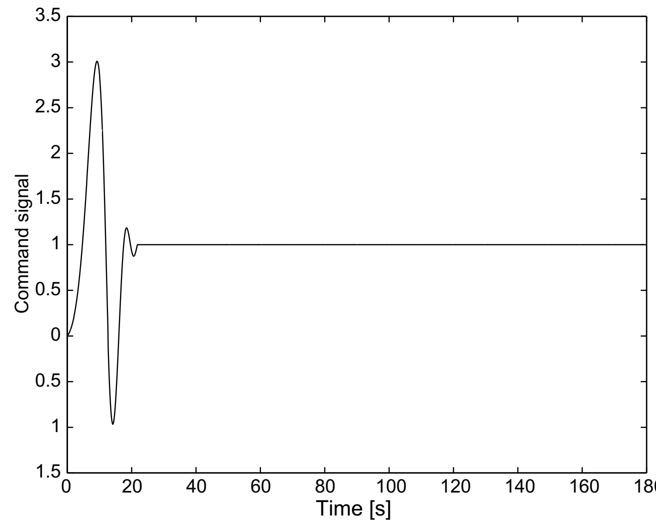

Fig. 17. The determined command signal for the level control experiment.

is very small with respect to its dominant time constant, in order to provide a significant result, a time delay of 10 s has been added via software at the plant input.

Despite the model is nonlinear (as the flow rate out of the tank depends on the square root of the level), a FOPDT model has been estimated by applying the area method to the open-loop response with a step from 2 V to 2.5 V at the process input. The FOPDT model obtained is

$$P_6(s) = \frac{1.98}{29s+1}e^{-11s}$$

Based on this model, it has been set $K_p = 1.24$, $T_i = 31$ and $T_d = 0$. The derivative action has not been employed due to the excessive measurement noise. In order to obtain a process output transition from 2 V to 3 V (starting when the process is at the steady-state), which corresponds approximately to a level transition from 6 cm to 12 cm, it has been

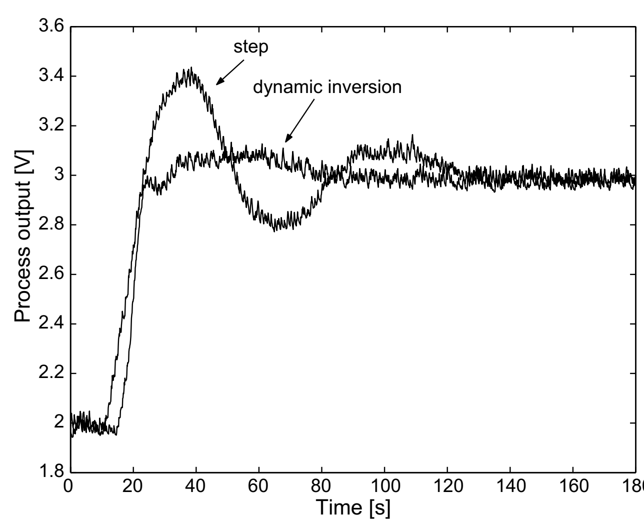

Fig. 18. The resulting process outputs with the new and the classic method for the level control experiment.

fixed $\tau = 10$ s and the inversion-based command signal has been determined. It is shown in Fig. 17. The corresponding process output, together with the step response is plotted in Fig. 18. A significant reduction of the overshoot and of the settling time appears. It turns out that these experimental results confirms the effectiveness of the methodology.

7. Conclusions

A noncausal approach for the improvement of the set-point following performances of PID controllers has been proposed. It is based on an input–output inversion procedure that allows to determine a closed-form expression of the command signal to be applied to the closed-loop system. The main merit of the methodology is that it provides predefined (high) performances almost independently from the PID parameters and from the adopted identification method. The unique design parameter (i.e. the desired transition time τ) has a clear physical meaning and it allows the user to handle the trade-off between aggressiveness, robustness and control activity. Thus, the technique integrates those features that are desirable in the industrial context and it appears to be suitable to implement in Distributed Control Systems as well as in single-station controllers.

Appendix A. Closed-form expressions

The closed-form expressions of the polynomials appearing in Eqs. (18)–(21) are given hereafter. First, consider the part of the inverse transfer function (13) related to the unstable zeros given by the Padè approximation, which can be written as

$$H_0^+(s) = \frac{d_1 s + d_0}{(s - z_R^+)^2 + z_I^{+2}}. \tag{34}$$

Then, we obtain (see Proposition 1):

$$p_1^+(\tau) = \frac{-48 d_0 z_R^+ z_I^{+3} - 12 d_1 z_I^{+5} + 36 d_1 z_I^+ z_R^{+4} + 24 d_1 z_I^{+3} z_R^{+2} + 48 d_0 z_R^{+3} z_I^+}{(z_R^{+2} + z_I^{+2})^4 z_I^+} + \frac{6 d_0 z_I^{+5} - 24 d_1 z_I^{+3} z_R^{+3} - 12 d_1 z_I^{+5} z_R^{-} 12 d_0 z_I^{+3} z_R^{+2} - 18 d_0 z_I^+ z_R^{+4} - 12 d_1 z_I^+ z_R^{+5}}{(z_R^{+2} + z_I^{+2})^4 z_I^+}\tau \tag{35}$$

$$2^+(\tau) = \frac{-12 d_1 z_R^{+5} - 12 d_0 z_R^{+4} + 72 d_0 z_I^{+2} z_R^{+2} - 12 d_0 z_I^{+4} + 24 d_1 z_I^{+2} z_R^{+3} + 36 d_1 z_I^{+4} z_R^+}{(z_R^{+2} + z_I^{+2})^4 z_I^+} + \frac{-12 d_0 z_R^{+3} z_I^{+2} + 6 d_0 z_R^{+5} + 6 d_1 z_R^{+6} + 6 d_1 z_I^{+2} z_R^{+4} - 6 d_1 z_I^{+4} z_R^{+2} - 18 d_0 * z_R^+ z_I^{+4} - 6 d_1 z_I^{+6}}{(z_R^{+2} + z_I^{+2})^4 z_I^+}\tau \tag{36}$$

$$s_0^+(t) = \frac{-6(6 d_1 z_R^{+4} - 2 d_1 z_I^{+4} + 8 d_0 z_R^{+3} + 4 d_1 z_I^{+2} z_R^{+2} - 8 d_0 z_I^{+2} z_R^+)}{z_R^{+8} + 4 z_R^{+6} z_I^{+2} + 6 z_R^{+4} z_I^{+4} + 4 z_R^{+2} z_I^{+6} z_I^{+8}} + \frac{-6(4 d_0 z_I^{+2} z_R^{+2} + 4 d_1 z_R^{+5} + 6 d_0 z_R^{+4} - 2 d_0 z_I^{+4} + 8 d_1 z_R^{+3} z_I^{+2} + 4 d_1 z_R^+ z_I^{+4})}{z_R^{+8} + 4 z_R^{+6} z_I^{+2} + 6 z_R^{+4} z_I^{+4} + 4 z_R^{+2} z_I^{+6} + z_I^{+8}} t + \frac{4 d_0 z_R^{+3} z_I^{+2} + 2 d_0 z_R^+ z_I^{+4} + d_1 z_R^{+6} + 3 d_1 z_R^{+4} z_I^{+2} + 3 d_1 z_R^{+2} z_I^{+4} + d_1 z_I^{+6} + 2 d_0 z_R^{+5}}{z_R^{+8} + 4 z_R^{+6} z_I^{+2} + 6 z_R^{+4} z_I^{+4} + 4 z_R^{+2} z_I^{+6} + z_I^{+8}} t^2 \tag{37}$$

$$s_1^+(t) = \frac{-6(-2 d_1 z_R^{+5} - 2 d_0 z_I^{+2} z_R^{+2} - 4 d_1 z_I^{+2} z_R^{+3} - 2 d_1 z_I^{+4} z_R^+ - 3 d_0 z_R^{+4} + d_0 z_I^{+4})}{z_R^{+8} + 4 z_R^{+6} z_I^{+2} + 6 z_R^{+4} z_I^{+4} + 4 z_R^{+2} z_I^{+6} + z_I^{+8}} + \frac{-6(-2 d_0 z_R^{+5} - 4 d_0 z_R^{+3} z_I^{+2} - 2 d_0 z_R^+ z_I^{+4} - d_1 z_R^{+6} - 3 d_1 z_R^{+4} z_I^{+2} - d_1 z_I^{+6} - 3 d_1 z_I^{+4} z_R^{+2})}{z_R^{+8} + 4 z_R^{+6} z_I^{+2} + 6 z_R^{+4} z_I^{+4} + 4 z_R^{+2} z_I^{+6} + z_I^{+8}} t \tag{38}$$

$$q_1^+(\tau) = \frac{12 d_1 z_I^{+5} - 36 d_1 z_I^+ z_R^{+4} + 48 d_0 z_R^+ z_I^{+3} - 48 d_0 z_R^{+3} z_I^+ - 24 d_1 z_I^{+3} z_R^{+2}}{z_I^+ z_R^{+8} + 4 z_I^{+3} z_R^{+6} + 6 z_I^{+5} z_R^{+4} + 4 z_I^{+7} z_R^{+2} + z_I^{+9}} + \frac{-12 d_1 z_I^{+5} z_R^+ - 24 d_1 z_I^{+3} z_R^{+3} + 6 d_0 z_I^{+5} - 12 d_0 z_I^{+3} z_R^{+2} - 12 d_1 z_I^+ z_R^{+5} - 18 d_0 z_I^+ z_R^{+4}}{z_I^+ z_R^{+8} + 4 z_I^{+3} z_R^{+6} + 6 z_I^{+5} z_R^{+4} + 4 z_I^{+7} z_R^{+2} + z_I^{+9}}\tau \tag{39}$$

$$q_2^+(\tau) = \frac{-72 d_0 z_I^{+2} z_R^{+2} - 36 d_1 z_I^{+4} z_R^+ - 24 d_1 z_I^{+2} z_R^{+3} + 12 d_0 z_R^{+4} + 12 d_0 z_I^{+4} + 12 d_1 z_R^{+5}}{z_I^+ z_R^{+8} + 4 z_I^{+3} z_R^{+6} + 6 z_I^{+5} z_R^{+4} + 4 z_I^{+7} z_R^{+2} + z_I^{+9}} + \frac{6 d_1 z_I^{+2} z_R^{+4} - 12 d_0 z_R +^3 z_I^{+2} + 6 d_0 z_R^{+5} + 6 d_1 z_R^{+6} - 6 d_1 z_I^{+6} - 6 d_1 z_I^{+4} z_R^{+2} - 18 d_0 z_R^+ z_I^{+4}}{z_I^+ z_R^{+8} + 4 z_I^{+3} z_R^{+6} + 6 z_I^{+5} z_R^{+4} + 4 z_I^{+7} z_R^{+2} + z_I^{+9}}\tau \tag{40}$$

With respect to the stable zeros, three cases have to be considered, related to the presence of two different real zeros, two coincident real zeros and two complex conjugate zeros, depending of the selected tuning of the controller. In the first case, the part of the inverse transfer function (13) related to the stable zeros can be written as

$$H_0^-(s) = \frac{c_1 s + c_0}{(s - z_1^-)(s - z_2^-)}. \tag{41}$$

Then, we obtain (see Proposition 1):

$$p_1^-(\tau) = \frac{-12c_0 {z_2^-}^4 - 12c_1 z_1^- {z_2^-}^4}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4} + \frac{6c_1 {z_1^-}^2 {z_2^-}^4 + 6c_0 z_1^- {z_2^-}^4}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4}\tau \tag{42}$$

$$p_2^-(\tau) = \frac{12c_1 {z_1^-}^4 z_2^- + 12c_0 {z_1^-}^4}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4} + \frac{-6c_0 {z_1^-}^4 z_2^- - 6c_1 {z_1^-}^4 {z_2^-}^2}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4}\tau \tag{43}$$

$$s_0^+(t) = \frac{-6(2{z_1^-}^3 c_0 + 2z_1^- {z_2^-}^2 c_0 + 2{z_1^-}^3 z_2^- c_1 + 2z_1^- {z_2^-}^3 c_1 + 2{z_1^-}^2 z_2^- c_0 + 2{z_1^-}^2 {z_2^-}^2 c_1 + 2{z_2^-}^3 c_0)}{{z_1^-}^4 {z_2^-}^4} + \frac{-6(2{z_1^-}^3 c_1 {z_2^-}^2 + 2{z_1^-}^2 c_1 {z_2^-}^3 + 2{z_1^-}^3 c_0 z_2^- + 2z_1^- c_0 {z_2^-}^3 + 2{z_1^-}^2 c_0 {z_2^-}^2)}{{z_1^-}^4 {z_2^-}^4} t + \frac{-6({z_1^-}^3 c_1 {z_2^-}^3 + {z_1^-}^3 c_0 {z_2^-}^2 + {z_1^-}^2 c_0 {z_2^-}^3)}{{z_1^-}^4 {z_2^-}^4} t^2 \tag{44}$$

$$s_1^+(t) = \frac{-6(-{z_1^-}^3 z_2^- c_0 - {z_1^-}^2 {z_2^-}^2 c_0 - z_1^- {z_2^-}^3 c_0 - {z_1^-}^2 {z_2^-}^3 c_1 - {z_1^-}^3 {z_2^-}^2 c_1)}{{z_1^-}^4 {z_2^-}^4} + \frac{-6(-{z_1^-}^3 c_1 {z_2^-}^3 - {z_1^-}^2 c_0 {z_2^-}^3 - {z_1^-}^3 c_0 {z_2^-}^2)}{{z_1^-}^4 {z_2^-}^4} t \tag{45}$$

$$q_1^+(\tau) = \frac{12c_0 {z_2^-}^4 + 12c_1 z_1^- {z_2^-}^4}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4} + \frac{6c_1 {z_1^-}^2 {z_2^-}^4 + 6c_0 z_1^- z_2^4}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4}\tau \tag{46}$$

$$q_2^+(\tau) = \frac{-12c_0 {z_1^-}^4 - 12c_1 {z_1^-}^4 z_2^-}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4} + \frac{-6c_1 {z_1^-}^4 {z_2^-}^2 - 6c_0 {z_1^-}^4 z_2^-}{{z_1^-}^4 (z_1^- - z_2^-){z_2^-}^4}\tau \tag{47}$$

In case the tuning of the controller is such that two coincident (stable) real zeros result, we can write

$$H_0^-(s) = \frac{c_1 s + c_0}{(s - z^-)^2}. \tag{48}$$

Then, it results:

$$p_1^-(\tau) = \frac{48c_0 + 36z^- c_1}{{z^-}^5} + \frac{-18c_0 z^- - 12c_1 {z^-}^2}{{z^-}^5}\tau \tag{49}$$

$$p_2^-(\tau) = \frac{-12c_0 z^- - 12c_1 {z^-}^2}{{z^-}^5} + \frac{6c_1 {z^-}^3 + 6c_0 {z^-}^2}{{z^-}^5}\tau \tag{50}$$

$$s_0^+(t) = \frac{-6(8c_0 + 6z^- c_1)}{{z^-}^5} + \frac{-6(6c_0 z^- + 4c_1 {z^-}^2)}{{z^-}^5} t + \frac{-6(2c_0 {z^-}^2 + c_1 {z^-}^3)}{{z^-}^5} t^2 \tag{51}$$

$$s_1^+(t) = \frac{-6(-3c_0 z^- - 2c_1 {z^-}^2)}{{z^-}^5} + \frac{-6(-2c_0 {z^-}^2 - c_1 {z^-}^3)}{{z^-}^5} t \tag{52}$$

$$q_1^+(\tau) = \frac{6(-8c_0 - 6z^- c_1)}{{z^-}^5} + \frac{6(-5c_0 z^- - 4c_1 {z^-}^2)}{{z^-}^5}\tau + \frac{6(-c_0 {z^-}^2 - {z^-}^3 c_1)}{{z^-}^5}\tau^2 \tag{53}$$

$$q_2^+(\tau) = \frac{6(2c_0 z^- + 2c_1 {z^-}^2)}{{z^-}^5} + \frac{6(c_0 {z^-}^2 + c_1 {z^-}^3)}{{z^-}^5}\tau \tag{54}$$

Finally, if the PID controller is tuned such as two complex conjugate zeros are posed, it can be written

$$H_0^-(s) = \frac{c_1 s + c_0}{(s - z_R^-)^2 + {z_I^-}^2}. \tag{55}$$

Then, the resulting polynomials are the same of expressions (35)–(40) where d_1, d_0 and z_R^+ have to be substituted with c_1, c_0 and z_R^-, respectively.

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