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Automatica 41 (2005) 305–313
automatica
www.elsevier.com/locate/automatica

Brief paper

Using stable input–output inversion for minimum-time feedforward constrained regulation of scalar systems☆

Aurelio Piazzi[a,*], Antonio Visioli[b]

[a]Dipartimento di Ingegneria dell'Informazione, University of Parma, Parco Area delle Science 181/a I-43100 Parma, Italy
[b]Dipartimento di Elettronica per l'Automazione, University of Brescia, Italy

Received 9 January 2004; received in revised form 7 June 2004; accepted 21 October 2004

Abstract

The paper approaches the feedforward minimum-time smooth control of non-minimum-phase linear scalar systems for set-point regulation. The aim is to synthesize a bounded input subject to non-saturating constraints on the input and its derivatives till a prespecified arbitrary order. The provided solution relies on a stable input–output dynamic inversion technique and an ad hoc parameterized family of output transition functions. The synthesized minimum-time input that exhibits pre- and post-actuation is determined by means of easily-implementable closed-form expressions. Examples are given to illustrate the methodology.

Keywords: Dynamic inversion; Set-point constrained regulation; Smooth control; Minimum-time; Linear systems

1. Introduction

It is common knowledge that a feedforward controller can significantly improve the performances of a feedback control system for set-point regulation. When preview information is available so that the next desired constant output is known in advance, a non-causal feedforward controller may be set out. This corresponds to the synthesis of a command input with pre-actuation by using stable dynamic inversion procedures (see for example Devasia, Chen, & Paden, 1996 and Hunt, Meyer, & Su, 1996 for the nonlinear and linear case, respectively, Zou & Devasia, 2004 and references therein contained for possible applications of the approach, whilst Benosman & Le Vey, 2003 and references therein contained focus on input–output inversion methods). In a recent paper, Perez and Devasia (2003) have shown how to synthesize an optimal minimum-energy input for the output transition between two given output values. Their solution is based on a stable inversion technique with pre- and post-actuation and could be readily adopted to improve the performances of any set-point feedback regulator.

In this paper, we deal with the synthesis of a feedforward smooth input for the set-point constrained regulation of a linear scalar system. Our aim is on determining a bounded, arbitrarily smooth input with non-saturating constraints on the input and its derivatives till a pre-specified order (note that this is not achieved with the classical (bang–bang) time-optimal approach). Our approach, which extends to non-minimum-phase systems in an earlier work (Piazzi & Visioli, 2001a), requires the adoption of an ad hoc parameterized family of output transition functions that allows to solve analytically a stable input–output dynamic inversion problem (see Section 3). Then, the minimization of the output transition time subject to all the given constraints can be posed and promptly solved (see Section 4). The resulting minimum-time input function could be then used to achieve high-performance set-point regulation in presence of

☆This paper was not presented at any IFAC meeting. This paper was recommended for publication in the revised form by Associate Editor F. Jabbari under the direction of Editor R. Tempo.

* Corresponding author. Tel.: +39 0521 905733; fax: +39 0521 905723.
E-mail addresses: aurelio@ce.unipr.it (A. Piazzi), antonio.visioli@ing.unibs.it (A. Visioli).

0005-1098/$ - see front matter
doi:10.1016/j.automatica.2004.10.009

non-saturating input constraints, such as slew-rate control constraints, that are important in a variety of applications.

The paper is organized as follows. Section 2 introduces within a behavioral framework the stable input–output dynamic inversion problem and the corresponding minimum-time input synthesis problem. Using transfer function techniques, Section 3 solves the stable inversion and provides a closed-form expression of the synthesized bounded input. It is also shown that this input converges to the desired output signal (apart a scaling factor) when the transition time approaches to infinity (see Proposition 6 and Remark 2). This property is actually crucial in showing that the addressed optimization problem admits a solution under very mild conditions (Proposition 8 reported in Section 4). Illustrative examples are presented in Section 5. The paper ends with the final remarks of Section 6.

Notation. $\mathbb{R}_+$ and $\mathbb{C}_+$ denote the set of positive real number and the set of complex number with positive real part respectively. C^i denotes the set of scalar real functions that are continuous till the ith derivative. The ith-order differential operator is D^i. The L_∞ norm of a real signal $f(t)$ defined and bounded over $(-\infty, +\infty)$ be $\|f(\cdot)\|_\infty := \sup_{t\in\mathbb{R}} |f(t)|$. If $p(\tau)$ is a polynomial with argument τ then $\deg\{p(\tau)\}$ denotes its degree and if $\deg\{p(\tau)\} \leqslant k$ we may alternatively denote this polynomial by $p[k](\tau)$.

2. Preliminaries and problem statements

Consider a general non-minimum-phase nth-order linear scalar system Σ whose transfer function is $(K_1 \neq 0)$

$$H(s) = K_1 \frac{b(s)}{a(s)} = K_1 \frac{s^m + b_{m-1}s^{m-1} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0}. \quad (1)$$

The polynomials $a(s)$ and $b(s)$ are coprime (no pole-zero cancellations occur) and the numerator of $H(s)$ can be factorized as

$$b(s) = b^-(s)b^+(s), \quad (2)$$

where $b^-(s)$ and $b^+(s)$ denote the polynomials associated to the zeros of Σ with negative and positive real part, respectively (Σ has not purely imaginary zeros). The input and output of Σ are $u \in \mathbb{R}$ and $y \in \mathbb{R}$, respectively and the relative order (or relative degree) of Σ is $\rho := n - m$. The set of all cause/effect pairs associated with Σ is denoted by

$$\begin{aligned}\mathscr{B} := \{(u(\cdot), y(\cdot)) \in P_c \times P_c : D^n y + a_{n-1}D^{n-1}y \\ + \cdots + a_0 y = K_1(D^m u + b_{m-1}D^{m-1}u \\ + \cdots + b_0 u)\},\end{aligned} \quad (3)$$

where P_c denotes the set of piecewise continuous functions defined over $(-\infty, +\infty)$, i.e. the real field $\mathbb{R}$. In the framework of the behavioral approach, $\mathscr{B}$ is the behavior set of Σ that can be rigorously introduced by means of the so-called *weak solutions* of the differential equation associated to Σ (Polderman & Willems, 1998).

The authors have shown in Piazzi and Visioli (2001a–c) that, for set-point regulation problems, a good choice for the output time function is the following *transition function* $y(t;\tau)$ defined as

$$y(t;\tau) = \begin{cases} 0 & \text{if } t \leqslant 0, \\ \dfrac{\int_0^t v^k(\tau - v)^k \, dv}{\int_0^\tau v^k(\tau - v)^k \, dv} & \text{if } 0 \leqslant t \leqslant \tau, \\ 1 & \text{if } t \geqslant \tau. \end{cases} \quad (4)$$

This transition function is actually the solution to a polynomial interpolation problem with end-point conditions $D^i y(0;\tau) = 0$, $i = 0, 1, \ldots, k$ and $y(\tau;\tau) = 1$, $D^i y(\tau;\tau) = 0$, $i = 1, 2, \ldots, k$. Therefore, function $y(t;\tau)$, parameterized by the transition time τ is a C^k-function over $(-\infty, +\infty)$ and is strictly increasing in the interval $[0, \tau]$ so that no overshooting nor undershooting appear in this output planning for set-point regulation. A closed-form expression for the transition function $y(t;\tau)$ over the time interval $[0, \tau]$ is given by (Piazzi & Visioli, 2001a):

$$y(t;\tau) = \frac{(2k+1)!}{k!\tau^{2k+1}} \sum_{i=0}^{k} \frac{(-1)^{k-i}}{i!(k-i)!(2k-i+1)} \tau^i t^{2k-i+1}. \quad (5)$$

Once $y(t;\tau)$ is defined according to (4) or (5), the following stable input–output (dynamic) inversion (SIOI) problem arises.

SIOI problem. Determine an input function $u(t;\tau)$ bounded over $(-\infty, +\infty)$ such that

$$(u(\cdot;\tau), y(\cdot;\tau)) \in \mathscr{B}. \quad (6)$$

An appropriate choice for the transition time may be done by solving the minimum-time input synthesis (MTIS) problem:

$$\min_{\tau\in\mathbb{R}^+} \tau$$

such that, $i = 0, 1, \ldots, l$,

$$|D^i u(t;\tau)| \leqslant u_M^{(i)}, \quad \forall t \in (-\infty, +\infty). \quad (7)$$

The positive values $u_M^{(i)}$, $i = 0, 1, \ldots, l$, are given bounds of the problem. In choosing the index l for the MTIS problem, the following proposition is useful (Polderman & Willems, 1998).

Proposition 1. *Consider any pair* $(u(\cdot), y(\cdot)) \in \mathscr{B}$. *Then,* $u(\cdot) \in C^l(\mathbb{R})$ *if and only if* $y(\cdot) \in C^{\rho+l}(\mathbb{R})$ *with* l *being a non-negative integer.*

Remark 1. The statement of Proposition 1 also holds with $l = -1$ provided that conventionally $C^{-1}(\mathbb{R})$ coincides with P_c, i.e. the set of piecewise continuous functions defined over $\mathbb{R}$.

3. Stable input–output inversion

By polynomial division, the inverse of the transfer function (1) can be expressed as

$$H^{-1}(s) = \frac{1}{K_1}\frac{a(s)}{b(s)} = \gamma_\rho s^\rho + \gamma_{\rho-1}s^{\rho-1} + \cdots + \gamma_0 + H_0(s), \tag{8}$$

where the strictly proper $H_0(s)=c(s)/b(s)$, $\deg\{c(s)\}\leqslant m-1$, represents the zero dynamics of Σ. Taking into account the factorization of $b(s)$ (see (2)) and by using the partial-fraction expansion, $H_0(s)$ can be decomposed into stable and unstable parts according to

$$H_0(s) = H_0^-(s) + H_0^+(s) = \frac{d(s)}{b^-(s)} + \frac{e(s)}{b^+(s)}, \tag{9}$$

where having defined $m^- := \deg\{b^-(s)\}$, $m^+ := \deg\{b^+(s)\}$, we have $\deg\{d(s)\}\leqslant m^- - 1$ and $\deg\{e(s)\}\leqslant m^+ - 1$.

The Laplace transform of the transition function be denoted by $Y(s;\tau) := \mathscr{L}[y(t;\tau)]$; then an input signal that causes the desired $y(t;\tau)$ on the output of Σ can be determined by the standard input–output dynamic inversion:

$$u_u(t;\tau) := \mathscr{L}^{-1}[H^{-1}(s)Y(s;\tau)]. \tag{10}$$

If $y(t;\tau)$ is sufficiently smooth, for example $k\geqslant\rho-1$, then evidently $(u_u(\cdot;\tau), y(\cdot,\tau)) \in \mathscr{B}$ but unfortunately $u_u(t;\tau)$ is unbounded over $[0,+\infty)$ due to the presence of the unstable zero dynamics. Nevertheless, the detailed knowledge of the structure of $u_u(t;\tau)$ is a key to solving the SIOI problem.

Define $\eta_0^-(t) := \mathscr{L}^{-1}[H_0^-(s)]$ and $\eta_0^+(t) := \mathscr{L}^{-1}[H_0^+(s)]$ and from (8)–(10) it follows that $u_u(t;\tau)=0$ when $t\in(-\infty,0)$ and if $t\geqslant 0$:

$$\begin{aligned} u_u(t;\tau) &= \gamma_\rho D^\rho y(t;\tau) + \gamma_{\rho-1}D^{\rho-1}y(t;\tau) + \cdots + \gamma_0 y(t;\tau) \\ &\quad + \int_0^t \eta_0^-(t-v)y(v;\tau)\,\mathrm{d}v + \int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v. \end{aligned} \tag{11}$$

The modes associated to $b^-(s)$ and $b^+(s)$ be denoted by $m_i^-(t)$, $i=1,\ldots,m^-$, and by $m_i^+(t)$, $i=1,\ldots,m^+$, respectively. Note that if z is a real zero of $b^-(s)$ or $b^+(s)$ with multiplicity h, then the associated modes are defined by

$$\{\mathrm{e}^{zt}, t\mathrm{e}^{zt}, \ldots, t^{h-1}\mathrm{e}^{zt}\}, \tag{12}$$

whereas if $\sigma\pm \mathrm{j}\omega$ are complex zeros of $b^-(s)$ or $b^+(s)$ with multiplicity h, then the associated modes are

$$\{\mathrm{e}^{\sigma t}\sin\omega t, \mathrm{e}^{\sigma t}\cos\omega t, t\mathrm{e}^{\sigma t}\sin\omega t, t\mathrm{e}^{\sigma t}\sin\omega t, \ldots, t^{h-1}\mathrm{e}^{\sigma t}\sin\omega t, t^{h-1}\mathrm{e}^{\sigma t}\cos\omega t\}. \tag{13}$$

Taking into account the polynomial expression (5) and the introduced modes (12) and (13), detailed closed-form expressions can be given for the convolution integrals appearing in (11).

Proposition 2.

$$\begin{aligned} \int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v &= H_0^+(0)y(t;\tau) + \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} p_i^+[k](\tau)m_i^+(t) \\ &\quad + \frac{1}{\tau^{2k+1}}T_0^+(t,\tau), \end{aligned} \tag{14}$$

where

$$T_0^+(t,\tau) = \begin{cases} \sum_{i=0}^{k} s_i^+[2k-i](t)\tau^i & if\ t\in[0,\tau] \\ \sum_{i=1}^{m^+} q_i^+[k](\tau)m_i^+(t-\tau) & if\ t>\tau \end{cases} \tag{15}$$

and $p_i^+[k](\tau)$, $q_i^+[k](\tau)$, $i=1,\ldots,m^+$ are τ-polynomials with degrees not exceeding k and $s_i^+[2k-i](t)$, $i=0,1,\ldots,k$ are t-polynomials with degrees ranging from $2k$ to k.

Proof. For brevity we consider the case where all the unstable zeros are distinct and real so that

$$\eta_0^+(t) = \sum_{i=1}^{m^+} g_i \mathrm{e}^{z_i t}, \quad z_i\in\mathbb{R}_+,\ \ i=1,\ldots,m^+.$$

The convolution integral (14) can be evaluated as the sum of single-modes integrals:

$$\int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v = \sum_{i=1}^{m^+} g_i\mathrm{e}^{z_i t}\int_0^t \mathrm{e}^{-z_i v}y(v;\tau)\,\mathrm{d}v. \tag{16}$$

When $t\in[0,\tau]$ the above expression becomes (see (5))

$$\sum_{i=1}^{m^+} g_i\mathrm{e}^{z_i t}\frac{(2k+1)!}{k!\tau^{2k+1}}\sum_{r=0}^{k}\frac{(-1)^{k-r}\tau^r}{r!(k-r)!(2k-r+1)} \times \int_0^t \mathrm{e}^{-z_i v}v^{2k-r+1}\,\mathrm{d}v. \tag{17}$$

Take $h\in\mathbb{N}$, then by mathematical induction we prove easily

$$\int_0^t \mathrm{e}^{-z_i v}v^h\,\mathrm{d}v = \frac{h!}{z_i^{h+1}} - \frac{\mathrm{e}^{-z_i t}}{z^{h+1}}\sum_{j=0}^{h}\frac{h!}{j!}z_i^j t^j. \tag{18}$$

Use the above formula (18) in the evaluation of expression (17) and after some algebraic rearranging we obtain

$$\begin{aligned}\int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v &= -\sum_{i=1}^{m^+}\frac{g_i}{z_i}\cdot\frac{(2k+1)!}{k!\tau^{2k+1}}\sum_{r=0}^{k}\frac{(-1)^{k-r}\tau^r t^{2k-r+1}}{r!(k-r)!(2k-r+1)}\\ &+\frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+}\sum_{r=0}^{k}\frac{g_i(2k+1)!(-1)^{k-r}(2k-r)!\tau^r}{k!r!(k-r)!z_i^{2k-r+2}}\mathrm{e}^{z_i t}\\ &+\frac{1}{\tau^{2k+1}}\sum_{r=0}^{k}\sum_{j=0}^{2k-r}\sum_{i=1}^{m^+}\frac{g_i(2k+1)!(-1)^{k-r+1}(2k-r)!}{k!r!(k-r)!z_i^{2k-r+2-j}j!}\\ &\times t^j\tau^r. \end{aligned} \tag{19}$$

Considering that

$$H_0^+(s)=\sum_{i=1}^{m^+}\frac{g_i}{s-z_i}$$

and appropriately defining the polynomials $p_i^+[k](\tau)$, $i=1,\dots,m^+$ and $s_r^+[2k-r](t)$, $r=0,1,\dots,k$, we rewrite (19) as

$$\begin{aligned}\int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v &= H_0^+(0)y(t;\tau)+\frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+}p_i^+[k](\tau)\mathrm{e}^{z_i t}\\ &+\frac{1}{\tau^{2k+1}}\sum_{r=0}^{k}s_r^+[2k-r](t)\tau^r. \end{aligned} \tag{20}$$

Hence, the above expression proves Proposition 2 for the case $t\in[0,\tau]$.

For the case $t>\tau$ and starting from (16) we have

$$\begin{aligned}\int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v &= \sum_{i=1}^{m^+}g_i\mathrm{e}^{z_i t}\int_0^\tau \mathrm{e}^{-z_i v}y(v;\tau)\,\mathrm{d}v\\ &+\sum_{i=1}^{m^+}g_i\mathrm{e}^{z_i t}\int_\tau^t \mathrm{e}^{-z_i v}\,\mathrm{d}v. \end{aligned} \tag{21}$$

The evaluation of the first integral can be done as in the first part of the proof whereas the second integral admits an immediate solution. Hence, we obtain

$$\begin{aligned}&\int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v\\ &=\Bigg\{-\sum_{i=1}^{m^+}\frac{g_i}{z_i}\mathrm{e}^{z_i(t-\tau)}y(\tau;\tau)+\frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+}p_i^+[k](\tau)\mathrm{e}^{z_i t}\\ &\quad-\frac{(2k+1)!}{k!\tau^{2k+1}}\sum_{i=1}^{m^+}g_i\mathrm{e}^{z_i(t-\tau)}\\ &\quad\times\sum_{r=0}^{k}\frac{(-1)^{k-r}(2k-r)!\tau^r}{r!(k-r)!}\sum_{j=0}^{2k-r}\frac{\tau^j}{j!z_i^{2k-r+2-j}}\Bigg\}\\ &\quad+\Bigg\{-\sum_{i=1}^{m^+}\frac{g_i}{z_i}+\sum_{i=1}^{m^+}\frac{g_i}{z_i}\mathrm{e}^{z_i(t-\tau)}\Bigg\}. \end{aligned} \tag{22}$$

Taking into account that $y(t;\tau)=1\ \forall t\geqslant\tau$ and after some algebraic manipulations we obtain

$$\begin{aligned}&\int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v\\ &=H_0^+(0)y(t;\tau)\\ &\quad+\frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+}p_i^+[k](\tau)\mathrm{e}^{z_i t}\\ &\quad-\frac{(2k+1)!}{k!\tau^{2k+1}}\sum_{i=1}^{m^+}g_i\mathrm{e}^{z_i(t-\tau)}\\ &\quad\times\sum_{r=0}^{k}\sum_{j=0}^{k-r}\frac{(-1)^{k-r}(2k-r)!\tau^{r+j}}{r!(k-r)!j!z_i^{2k-r+2-j}}\\ &\quad-\frac{(2k+1)!}{k!\tau^{2k+1}}\sum_{i=1}^{m^+}g_i\mathrm{e}^{z_i(t-\tau)}\\ &\quad\times\sum_{r=0}^{k}\sum_{j=k-r+1}^{2k-r}\frac{(-1)^{k-r}(2k-r)!\tau^{r+j}}{r!(k-r)!j!z_i^{2k-r+2-j}}. \end{aligned} \tag{23}$$

The last addend of expression (23) is proven to be equal to zero. Indeed, note that by doing the variable change $s=r+j$ we have

$$\begin{aligned}&\sum_{r=0}^{k}\sum_{j=k-r+1}^{2k-r}\frac{(-1)^{k-r}(2k-r)!\tau^{r+j}}{r!(k-r)!j!z_i^{2k-r+2-j}}\\ &=\sum_{s=k+1}^{2k}\sum_{r=0}^{k}\frac{(-1)^{k-r}(2k-r)!}{r!(k-r)!(s-r)!}\frac{\tau^s}{z^{2k+2-s}} \end{aligned} \tag{24}$$

and, by mathematical induction, for $s=k+1,\dots,2k$

$$\sum_{r=0}^{k}\frac{(-1)^{k-r}(2k-r)!}{r!(k-r)!(s-r)!}=0. \tag{25}$$

Appropriately defining the polynomials $q_i^+[k](\tau)$, $i = 1, \ldots, m^+$, expression (23) becomes

$$\int_0^t \eta_0^+(t-v)y(v;\tau)\,\mathrm{d}v = H_0^+(0)y(t;\tau) + \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} p_i^+[k](\tau)\mathrm{e}^{z_i t} + \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} q_i^+[k](\tau)\mathrm{e}^{z_i(t-\tau)}. \quad (26)$$

Evidently, expression (26) that holds for $t > \tau$ coincides with formulae (14) and (15). This completes the proof. □

Proposition 3.

$$\int_0^t \eta_0^-(t-v)y(v;\tau)\,\mathrm{d}v = H_0^-(0)y(t;\tau) + \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^-} p_i^-[k](\tau)m_i^-(t) + \frac{1}{\tau^{2k+1}}T_0^-(t,\tau), \quad (27)$$

where

$$T_0^-(t,\tau) = \begin{cases} \sum_{i=0}^{k} s_i^-[2k-i](t)\tau^i & \text{if } t \in [0,\tau], \\ \sum_{i=1}^{m^-} q_i^-[k](\tau)m_i^-(t-\tau) & \text{if } t > \tau, \end{cases} \quad (28)$$

and $p_i^-[k](\tau)$, $q_i^-[k](\tau)$, $i = 1, \ldots, m^-$ *are* τ*-polynomials with degrees not exceeding* k *and* $s_i^-[2k-i](t)$, $i = 0, 1, \ldots, k$ *are* t*-polynomials with degrees ranging from* $2k$ *to* k.

Proof of the above proposition is similar to that of Proposition 2 and therefore it is omitted. Taking into account Proposition 2 we define, over the whole time axis, the "correcting" input $u_c(t;\tau)$ which is a suitable linear combination of the unstable zero modes of Σ:

$$u_c(t;\tau) := -\frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} p_i^+[k](\tau)m_i^+(t) - \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} q_i^+[k](\tau)m_i^+(t-\tau), \quad t \in (-\infty, +\infty). \quad (29)$$

Lemma 4. *Having defined the input* $u_c(t;\tau)$ *in* (29) *we have*

$$(u_c(\cdot;\tau), 0) \in \mathscr{B} \quad \forall \tau \in \mathbb{R}_+. \quad (30)$$

Proof. Omitted for brevity.

Solution to the SIOI problem is provided by the following input function

$$u(t;\tau) := u_u(t;\tau) + u_c(t;\tau), \quad t \in (-\infty, +\infty). \quad (31)$$

Taking into account Propositions 2 and 3, relation (11), and definition (29), we can deduce the closed-form expression of $u(t;\tau)$:

$$u(t;\tau) = -\frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} p_i^+[k](\tau)m_i^+(t) - \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} q_i^+[k](\tau)m_i^+(t-\tau) \quad \text{if } t < 0, \quad (32)$$

$$u(t;\tau) = \gamma_\rho D^\rho y(t;\tau) + \cdots + \gamma_0 y(t;\tau) + H_0(0)y(t;\tau) + \frac{1}{\tau^{2k+1}}\sum_{i=0}^{k}(s_i^+[2k-i](t) + s_i^-[2k-i](t))\tau^i - \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^+} q_i^+[k](\tau)m_i^+(t-\tau) + \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^-} p_i^-[k](\tau)m_i^-(t) \quad \text{if } t \in [0,\tau], \quad (33)$$

$$u(t;\tau) = \gamma_0 + H_0(0) + \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^-} p_i^-[k](\tau)m_i^-(t) + \frac{1}{\tau^{2k+1}}\sum_{i=1}^{m^-} q_i^-[k](\tau)m_i^-(t-\tau) \quad \text{if } t > \tau. \quad (34)$$

Defining the input according to (31) implies a mathematically perfect cancellation between the unbounded modes of the standard inverse input $u_u(t;\tau)$ and the (unbounded) zero modes of the correcting input $u_c(t;\tau)$. Moreover, note that for negative times the system Σ evolves on the so-called "zero dynamics" while keeping the output on the constant zero value.

Proposition 5 (*Solution to the SIOI problem*). *Assume* $k \geqslant \rho - 1$. *For any* $\tau > 0$ *function* $u(t;\tau)$ *defined by* (32)–(34) *is bounded over* $(-\infty, +\infty)$ *and* $(u(\cdot;\tau), y(\cdot;\tau)) \in \mathscr{B}$.

Proof. The assumption $k \geqslant \rho - 1$ ensures $y(\cdot;\tau) \in C^{\rho-1}$ and, by (11), $u_u(\cdot;\tau) \in C^{-1}$ so that $(u_u(\cdot;\tau), y(\cdot;\tau)) \in \mathscr{B}$. Then, definition (31) and Lemma 4 prove that $(u(\cdot;\tau), y(\cdot;\tau)) \in \mathscr{B}$ for any $\tau > 0$ by virtue of linear superposition. On the other hand $u(\cdot;\tau)$ is, by construction, bounded over $(-\infty, +\infty)$ because of the exact cancellation of all the unstable modes appearing in $u_u(\cdot,\tau)$ (see (32)–(34)). In particular, note that the unstable modes $m_i^+(t)$ and $m_i^+(t-\tau)$ are bounded functions over $(-\infty, 0]$. □

The provided solution $u(t;\tau)$ to the SIOI problem is a noncausal signal due to the presence of a linear combination of unstable modes over $(-\infty, 0]$. A relevant property of this solution is given by the next result.

Proposition 6. *Assume* $k \geqslant \rho - 1$. *The cause/effect pair* $(u(\cdot;\tau), y(\cdot;\tau))$ *defined by* (4) *and* (32)–(34) *satisfies the* *following limit*:

$$\lim_{\tau\to+\infty} \|H(0)u(\cdot;\tau) - y(\cdot;\tau)\|_\infty = 0. \tag{35}$$

A useful result in order to prove Proposition 6 is given by this Lemma (see Piazzi & Visioli, 2001a).

Lemma 7. *Let be given the transition signal* $y(\cdot;\tau) \in C^k(\mathbb{R})$ *defined in* (4). *Then, there exist constants* $c_{ki} \in \mathbb{R}_+$, $i = 1, \dots, k$ *such that*

$$\max_{t\in[0,\tau]} |D^i y(t;\tau)| = \frac{c_{ki}}{\tau^i}. \tag{36}$$

Proof of Proposition 6. An equivalent statement to (35) is

$$\lim_{\tau\to+\infty} H(0)u(t;\tau) = y(t;\tau) \tag{37}$$

uniformly over $t \in \mathbb{R}$. To prove this first note that

$$\lim_{\tau\to+\infty} u(t;\tau) = 0 \quad \text{uniformly over } t \in (-\infty, 0]. \tag{38}$$

Indeed, for any $\tau \in \mathbb{R}_+$, the unstable modes $m_i^+(t)$ and $m_i^+(t-\tau)$ are bounded over $(-\infty, 0]$ and evidently ($i = 1, \dots, m^+$)

$$\lim_{\tau\to+\infty} \frac{1}{\tau^{2k+1}} p_i^+[k](\tau) = 0, \qquad \lim_{\tau\to+\infty} \frac{1}{\tau^{2k+1}} q_i^+[k](\tau) = 0.$$

With an analogous reasoning, this time involving the stable modes $m_i^-(t)$ and $m_i^-(t-\tau)$, we prove

$$\lim_{\tau\to+\infty} u(t;\tau) = \gamma_0 + H_0(0) \quad \text{uniformly over } t \in [\tau, +\infty). \tag{39}$$

The third step focuses on expression (33) that gives $u(t;\tau)$ when $t \in [0,\tau]$. By virtue of Lemma 7 we have for $i = 1, \dots, k$

$$\lim_{\tau\to+\infty} D^i y(t;\tau) = 0 \quad \text{uniformly over } t \in [0,\tau]. \tag{40}$$

Then, note there exist constants $\zeta_i^+,\ \zeta_i^- \in \mathbb{R}_+$ such that

$$|m_i^+(t-\tau)| < \zeta_i^+, \quad \forall t \in [0,\tau] \text{ and } \forall \tau > 0,$$
$$|m_i^-(t)| < \zeta_i^-, \quad \forall t \in [0,\tau] \text{ and } \forall \tau > 0.$$

This implies

$$\lim_{\tau\to+\infty} \frac{1}{\tau^{2k+1}} \sum_{i=1}^{m^+} q_i^+[k](\tau) m_i^+(t-\tau) = 0,$$
$$\lim_{\tau\to+\infty} \frac{1}{\tau^{2k+1}} \sum_{i=1}^{m^-} p_i^+[k](\tau) m_i^-(t) = 0, \tag{41}$$

where both limits hold uniformly over $t \in [0,\tau]$. Moreover, it holds

$$\lim_{\tau\to+\infty} \frac{1}{\tau^{2k+1}} \sum_{i=1}^{k} s_i^+[2k-i](t)\tau^i = 0,$$
$$\lim_{\tau\to+\infty} \frac{1}{\tau^{2k+1}} \sum_{i=1}^{k} s_i^-[2k-i](t)\tau^i = 0 \tag{42}$$

again, uniformly over $t \in [0,\tau]$.

By gathering statements (40)–(42) we deduce

$$\lim_{\tau\to+\infty} u(t;\tau) = (\gamma_0 + H_0(0))y(t;\tau) \tag{43}$$

uniformly over $t \in [0,\tau]$.

Considering that $y(t;\tau) = 0$ if $t < 0$ and $y(t;\tau) = 1$ if $t > \tau$ we can restate, by virtue of (38) and (39), that limit (43) also holds uniformly over $t \in \mathbb{R}$. Finally observe that $\gamma_0 + H_0(0) = H^{-1}(0)$ so that (43) is equivalent to (37). This ends the proof. □

Remark 2. Roughly speaking, Proposition 6 points out that the synthesized input $u(t;\tau)$ coincides (apart the scaling factor $H(0)$) with the desired output signal $y(t;\tau)$ when the transition time τ approaches infinity. Hence, $u(t;\tau)$ converges to a causal signal, i.e. a signal that is identically zero over the negative time axis, for $\tau \to +\infty$.

4. Minimizing the transition time

Using the synthesized input $u(t;\tau)$ for the purpose of set-point constrained regulation (see Piazzi & Visioli, 2001b) requires imposing non-saturating bounds on $u(t;\tau)$ and its derivatives as shown by inequalities (7): $|D^i u(t;\tau)| \leqslant u_M^{(i)}$ $\forall t \in \mathbb{R}$, $i = 0, 1, \dots, l$. Typically, when slew rate control constraints are crucial, setting $l = 1$ suffices. However, in the general case, given any non-negative integer l and bounds $u_M^{(i)} \in \mathbb{R}_+$, $i = 0, 1, \dots, l$ the MTIS problem is on finding

$$\tau^* := \min\{\tau > 0 : |D^i u(t;\tau)| \leqslant u_M^{(i)}, \quad \forall t \in \mathbb{R},$$
$$i = 0, 1, \dots, l\}. \tag{44}$$

In order for the above problem (44) to be well-posed, $D^l u(t;\tau)$ should be a continuous function over $(-\infty, +\infty)$. By virtue of Proposition 1 and considering that $y(\cdot;\tau) \in C^k$ we have to impose that $k \geqslant \rho + l$. On the other hand, the next result evidences that conditions ensuring the existence of the optimal τ^* are quite mild.

Proposition 8. *The MTIS problem has a solution if*

$$u_M^{(0)} > \frac{1}{|H(0)|} \quad \textit{and} \quad u_M^{(i)} > 0, \quad i = 1, \dots, l. \tag{45}$$

Proof. The proof is based on showing that the feasible set of the MTIS problem is not empty provided that the conditions (45) are satisfied. Proposition 6 implies that $u(t;\tau)$

converges to $(1/H(0))y(t;\tau)$ uniformly over $t \in \mathbb{R}$ when $\tau \to +\infty$. Hence, for any $\varepsilon_0 \in \mathbb{R}_+$ there exists $\tau_0 \in \mathbb{R}_+$, depending on ε_0 only, such that

$$\left| u(t;\tau) - \frac{1}{H(0)}\, y(t;\tau) \right| \leqslant \varepsilon_0, \quad \forall t \in \mathbb{R},\ \forall \tau \geqslant \tau_0 \tag{46}$$

Choose $\varepsilon_0 := u_M^{(0)} - 1/|H(0)|$ that is positive by virtue of (45). Therefore for any $t \in \mathbb{R}$ and any $\tau \geqslant \tau_0$

$$\begin{cases} u(t;\tau) \leqslant u_M^{(0)} - \frac{1}{|H(0)|} + \frac{1}{H(0)} y(t;\tau) \\ -u_M^{(0)} + \frac{1}{|H(0)|} + \frac{1}{H(0)}\, y(t;\tau) \leqslant u(t;\tau). \end{cases}$$

Taking into account that $y(t;\tau) \in [0,1]$, $\forall t \in \mathbb{R}$ we obtain $u(t;\tau) \leqslant u_M^{(0)}$ and $-u_M^{(0)} \leqslant u(t;\tau)$ for all $t \in \mathbb{R}$ and all $\tau \geqslant \tau_0$. This means for all $\tau \geqslant \tau_0$:

$$|u(t;\tau)| \leqslant u_M^{(0)} \quad \forall t \in \mathbb{R}. \tag{47}$$

Considering the closed-form expression of $u(t;\tau)$ given by (32)–(34) we have ($i = 1,\ldots,l$)

$$\begin{aligned} D^i u(t;\tau) = & -\frac{1}{\tau^{2k+1}} \sum_{j=1}^{m^+} p_j^+[k](\tau) D^i m_j^+(t) \\ & -\frac{1}{\tau^{2k+1}} \sum_{j=1}^{m^+} q_j^+[k](\tau) D^i m_j^+(t-\tau) \quad \text{if } t<0, \end{aligned} \tag{48}$$

$$\begin{aligned} D^i u(t;\tau) = & \gamma_\rho D^{\rho+i} y(t;\tau) + \cdots + \gamma_0 D^i y(t;\tau) \\ & + H(0) D^i y(t;\tau) + \frac{1}{\tau^{2k+1}} \\ & \times \sum_{j=0}^{k} [D^i s_j^+[2k-j](t) \\ & + D^i s_j^-[2k-j](t)]\tau^i \\ & - \frac{1}{\tau^{2k+1}} \sum_{j=1}^{m^+} q_j^+[k](\tau) D^i m_j^+(t-\tau) \\ & + \frac{1}{\tau^{2k+1}} \sum_{j=1}^{m^-} p_j^-[k](\tau) D^i m_j^-(t) \quad \text{if } t \in [0,\tau], \end{aligned} \tag{49}$$

$$\begin{aligned} D^i u(t;\tau) = & \frac{1}{\tau^{2k+1}} \sum_{j=1}^{m^-} p_j^-[k](\tau) D^i m_j^-(t) \\ & + \frac{1}{\tau^{2k+1}} \sum_{j=1}^{m^-} q_j^-[k](\tau) D^i m_j^-(t-\tau) \quad \text{if } t>\tau. \end{aligned} \tag{50}$$

Taking into account Lemma 7 and that functions $D^i m_j^+(t)$, $D^i m_j^+(t-\tau)$ are bounded over $(-\infty,0]$, functions $D^i m_j^+(t-\tau)$, $D^i m_j^-(t)$ are bounded over $[0,\tau]$ uniformly for any $\tau \in \mathbb{R}_+$, and functions $D^i m_j^-(t)$, $D^i m_j^-(t-\tau)$ are bounded over $[\tau,+\infty)$ uniformly for any $\tau \in \mathbb{R}_+$ we can deduce that for any $\varepsilon_i \in \mathbb{R}_+$ there exists $\tau_i \in \mathbb{R}_+$, depending on ε_i only, such that ($i = 1,\ldots,l$)

$$|D^i u(t;\tau)| \leqslant \varepsilon_i \quad \forall t \in \mathbb{R} \text{ and } \forall \tau \geqslant \tau_i. \tag{51}$$

Choose $\varepsilon_i := u_M^{(i)}$ that is positive by conditions (45). Therefore, from relations (47) and (51) we infer that all the constraint inequalities of the MTIS problem are satisfied when $\tau > \min\{\tau_0,\tau_1,\ldots,\tau_l\}$:

$$|D^i u(t;\tau)| \leqslant u_M^{(i)} \quad \forall t \in \mathbb{R},\ i = 0,1,\ldots,l. \tag{52}$$

By noting that ($i = 0,1,\ldots,l$)

$$\lim_{\tau \to 0^+} \max_{t \in \mathbb{R}} |D^i u(t;\tau)| = +\infty \tag{53}$$

and $D^i(t;\tau)$ is bounded over $t \in \mathbb{R}$ for any $\tau \in \mathbb{R}_+$ and it is continuous with respect to both arguments $t \in \mathbb{R}$ and $\tau \in \mathbb{R}_+$ we deduce the existence of the optimal τ^*, solution of problem (44). □

It is worth noting that the sufficient conditions (45) are also necessary ones in practice. Indeed, if any of the conditions (45) fails to be satisfied, the feedforward constrained regulation of system (1) is almost impossible regardless of the chosen planned output transition function. For example, if $u_M^{(0)} < 1/|H(0)|$ the control input cannot sustain the constant desired output even in the system steady-state conditions.

A practical, approximate determination of the optimal τ^* can be easily pursued by setting out a bisection algorithm over the parameter τ in conjunction with a gridding of the time axis (see Piazzi & Visioli, 2001a). When a numerical, rigorous determination of τ^* is sought, problem (44) should be approached with the tools of global optimization. From this standpoint, interval analysis could be successfully adapted in order to determine τ^* with guaranteed lower and upper bounds (Hansen & Walster, 2003). This approach is indeed facilitated by the exact knowledge of $u(t;\tau)$ and its derivatives offered by the derived closed-form expressions (32)–(34) and (48)–(50).

5. Illustrative examples

From a practical point of view, in order to use the synthesized minimum-time input function (32)–(34), it is necessary to truncate it, resulting therefore in an approximate generation of the desired output $y(t;\tau^*)$. However, this can be done with arbitrary precision given any couple of small parameters $\varepsilon_0 > 0$ and $\varepsilon_1 > 0$. Indeed, compute

$$t_0 := \max\{t' \in \mathbb{R} : |u(t;\tau^*)| \leqslant \varepsilon_0 \quad \forall t \in (-\infty, t']\}$$

and define $t_s := \min\{0, t_0\}$. Similarly, compute

$$t_1 := \min \left\{ t' \in \mathbb{R} : \left| u(t;\tau^*) - \frac{1}{H_0(0)} \right| \leqslant \varepsilon_1 \ \ \forall t \in [t', \infty) \right\}.$$

and $t_f := \max\{\tau^*, t_1\}$. Thus, the approximate input signal to be actually used is

$$u_a(t;\tau^*) := \begin{cases} 0 & \text{for } t < t_s, \\ u(t;\tau^*) & \text{for } t_s \leqslant t \leqslant t_f, \\ \frac{1}{H_0(0)} & \text{for } t > t_f. \end{cases}$$

Note that typically $t_s < 0$ and $t_f > \tau^*$, associated to the pre- and post-action control, respectively.

As a first example we consider the following system, with one unstable zero, one stable zero and with a relative order $\rho = 1$:

$$H(s) = -4\,\frac{(s-1)(s+1)}{(s+2)(s^2+s+2)}. \tag{54}$$

Only a limit on the input amplitude is selected (i.e. $l = 0$), namely, $u_M^{(0)} = 1.2$. Consequently, it has been fixed $k = 1$ (see Section 4). By following the proposed methodology we obtain (see (8)):

$$H^{-1}(s) = -\frac{1}{4}\,s - 1 - \frac{1}{4}\,\frac{7s+8}{(s-1)(s+1)}. \tag{55}$$

By using the partial fraction expansion, we decompose $H_0(s)$ into the stable and unstable part (see (9)), thus obtaining:

$$H_0^-(s) = \frac{1/8}{s+1}, \quad H_0^+(s) = -\frac{15/8}{s-1} \tag{56}$$

and, by applying the inverse Laplace transform:

$$\eta_0^-(t) = \frac{1}{8}\,\mathrm{e}^{-t}, \quad \eta_0^+(t) = -\frac{15}{8}\,\mathrm{e}^{t}. \tag{57}$$

The modes associated with the stable and unstable zero are clearly $m_1^-(t) = \mathrm{e}^{-t}$ and $m_1^+(t) = \mathrm{e}^{t}$, respectively. Note that $m^+ = m^- = 1$. By choosing $k = 1$, the following transition function is obtained (see (5))

$$y(t;\tau) = -2\,\frac{t^3}{\tau^3} + 3\,\frac{t^2}{\tau^2}, \quad t \in [0,\tau]. \tag{58}$$

Then, the convolution integrals appearing in (11) can be evaluated by considering the resulting following expressions (see (14) and (15) and (27) and (28)):

$$\begin{aligned}
&p_1^+[1](\tau) = \frac{45}{2} - \frac{45}{4}\tau,\\
&s_0^+[2](t) = -\frac{45}{2} - \frac{45}{2}\,t - \frac{45}{4}\,t^2, \quad s_1^+[1](t) = \frac{45}{4} + \frac{45}{4}\,t,\\
&q_1^+[1](\tau) = -\frac{45}{2} - \frac{45}{4}\,\tau,\\
&p_1^-[1](\tau) = -\frac{3}{2} - \frac{3}{4}\,\tau,\\
&s_0^-[2](t) = \frac{3}{2} - \frac{3}{2}\,t + \frac{3}{4}\,t^2, \quad s_1^-[1](t) = \frac{3}{4} - \frac{3}{4}\,t,\\
&q_1^-[1](\tau) = +\frac{3}{2} - \frac{3}{4}\,\tau.
\end{aligned} \tag{59}$$

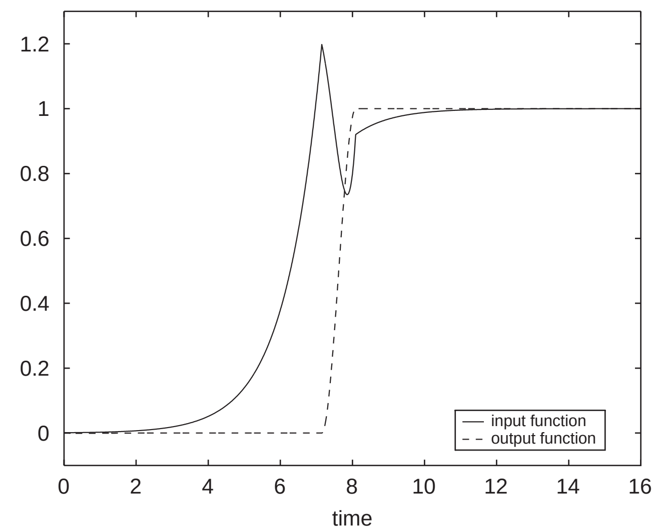

Fig. 1. Minimum-time functions for Example 1.

Hence, the expression of $u(t;\tau)$ is (see (32)–(34))

$$\begin{aligned} u(t;\tau) = &-\frac{1}{\tau^3}\left(\frac{45}{2} - \frac{45}{4}\tau\right)\mathrm{e}^{t} \\ &-\frac{1}{\tau^3}\left(-\frac{45}{2} - \frac{45}{4}\,\tau\right)\mathrm{e}^{t-\tau} \quad \text{if } t < 0, \end{aligned} \tag{60}$$

$$\begin{aligned} u(t;\tau) = &-\frac{1}{4}\left(-6\,\frac{t^2}{\tau^3} + 6\,\frac{t}{\tau^2}\right) - 1\left(-2\,\frac{t^3}{\tau^3} + 3\,\frac{t^2}{\tau^2}\right) \\ &+\frac{1}{\tau^3}\left(-\frac{45}{2} - \frac{45}{2}\,t - \frac{45}{4}\,t^2 + \frac{3}{2} - \frac{3}{2}\,t + \frac{3}{4}\,t^2\right) \\ &+\frac{1}{\tau^3}\left(\frac{45}{4} + \frac{45}{4}\,t + \frac{3}{4} - \frac{3}{4}\,t\right)\tau \\ &-\frac{1}{\tau^3}\left(-\frac{45}{2} - \frac{45}{4}\,\tau\right)\mathrm{e}^{t-\tau} \\ &+\frac{1}{\tau^3}\left(-\frac{3}{2} - \frac{3}{4}\,\tau\right)\mathrm{e}^{-t} \quad \text{if } t \in [0,\tau], \end{aligned} \tag{61}$$

$$\begin{aligned} u(t;\tau) = &-1 + 2 + \frac{1}{\tau^3}\left(-\frac{3}{2} - \frac{3}{4}\,\tau\right)\mathrm{e}^{-t} \\ &+\frac{1}{\tau^3}\left(\frac{3}{2} - \frac{3}{4}\,\tau\right)\mathrm{e}^{\tau-t} \quad \text{if } t > \tau. \end{aligned} \tag{62}$$

Taking into account that $H(0) = 1$ we have $u_M^{(0)} > 1/|H(0)|$ (see Proposition 8) so that the MTIS problem admits a solution. By applying a bisection procedure (see Piazzi & Visioli, 2001a) it results $\tau^* = 0.94$. Finally, fixing $\varepsilon_0 = \varepsilon_1 = 1 \times 10^{-4}$ it results $t_s = -7.15$ and $t_f = 7.35$. The resulting minimum-time input and output functions are reported in Fig. 1. Note that the time axis has been conveniently shifted in order to have $t_s = 0$.

As a second example, the following system is considered

$$H(s) = \frac{(s+1)^2((s-1)^2+4)^2}{(s^2+s+1)^2(s+2)(s+3)(s+5)}. \tag{63}$$

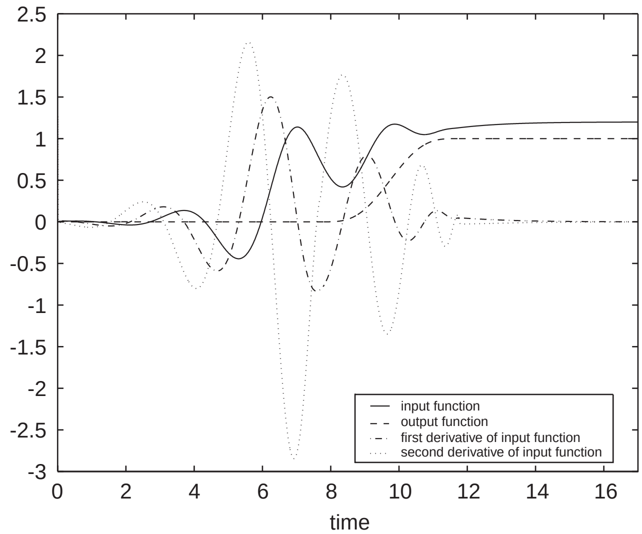

Fig. 2. Resulting functions for Example 2.

In this case bounds until the second derivative of the input function are selected (i.e. $l=2$): $u_M^{(0)}=1.2$, $u_M^{(1)}=1.5$ and $u_M^{(2)}=3$. As it is $\rho=1$, we selected $k=3$. For this case, details of the determination of the closed-form expression of the parameterized input function $u(t;\tau)$ are omitted for brevity. By solving the MTIS problem it results $\tau^*=4.08$. By choosing $\varepsilon_0=\varepsilon_1=0.005$ it results $t_s=-7.70$ and $t_f=7.40$. The minimum-time approximated input function $u_a(t;\tau^*)$ is plotted in Fig. 2, together with its first and second derivatives and with the resulting output function. Note again that the time axis has been properly shifted in order to have $t_s=0$.

6. Conclusions

The paper has presented an inversion-based solution to the minimum-time feedforward set-point regulation of scalar systems subject to non-saturating input constraints. This solution allows obtaining an arbitrarily smooth input that exhibits pre-action when unstable zeros are present and post-action when stable zeros are present (see (32)–(34)). A key point of the technique is the adoption of the "transition polynomials" to plan a non over- and non under-shooting output function (Piazzi & Visioli, 2001a). The method can be easily used to improve the performances of set-point feedback regulators especially in the mechatronic field (Benosman & Le Vey, 2003; Guarino Lo Bianco & Piazzi, 2002).

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Aurelio Piazzi received the Laurea degree in nuclear engineering in 1982 and the Ph.D. degree in system engineering in 1987, both from the University of Bologna, Italy. From 1990 to 1992, he was Research Associate in System Theory, DEIS, University of Bologna. Since 1992 he has been at the University of Parma where he is currently Full Professor of Automatic Control at the Department of Informatics Engineering. His main research interests are in system and control theory and related engineering applications. His recent research activities have focused on methods of global optimization applied to robotics and control problems and on dynamic inversion techniques for autonomous vehicle guidance and for the design of high-performance control systems. He has been the scientific coordinator of the European Project COOKIES in collaboration with Gruppo Colussi SpA (Perugia, Italy) for the vision-based control of industrial ovens. He is a member of IEEE and SIAM.

Antonio Visioli received the Laurea degree in electronic engineering from the University of Parma, Italy, and the Ph.D. degree in applied mechanics from the University of Brescia, Italy, in 1995 and 1999, respectively. His Ph.D. dissertation was on control strategies for industrial robot manipulators. He is currently an Assistant Professor of Automatic Control, Department of Electronics for Automation, University of Brescia, Italy. His research interests include industrial robot control and trajectory planning, dynamic-inversion-based control and process control. Dr. Visioli is member of IEEE, IFAC and Anipla (Italian Association for the Automation).