Optimal dynamic-inversion-based control of an overhead crane

A. Piazzi and A. Visioli

Abstract: A methodology is proposed to control the transient sway and residual oscillation of a payload carried by an overhead crane. The design approach is based on a linearised model of the crane and consists of dampening the linearised system by an observer-based controller and applying a dynamic inversion procedure in order to assure a predetermined oscillation free polynomial motion law for the payload. Polynomial functions are adopted in order to guarantee that the input function has a continuous derivative of an arbitrary order. Moreover, the motion time can be minimised, taking into account constraints on the actuators, by means of a simple bisection algorithm. Parameter uncertainties are taken into account during the whole design procedure. Simulation results, based on a nonlinear crane model, show how the method is also effective when the payload is hoisted or lowered during the motion, and when friction effects are considered.

1 Introduction

The safety and efficiency of the operation of an overhead crane are generally reduced by the transient sway and residual oscillation of either the empty hook or the payload. In general, this problem is tackled by the experience and skill of the operators, who try to impose a deceleration law that reduces the oscillation caused by the acceleration. Moreover, a man is often tasked to stop the hook or the payload. Thus, the performances of the system can be significantly improved by using an appropriate automatic control architecture, which is capable of reducing the swing effect and minimising the motion time in order to increase the throughput of the crane. However, the design of the controller is a challenging problem, since the system, which can be regarded as a single-pendulum, is nonlinear (and hence, if a linearised model is considered for the controller design, then the attained performances have to be verified against a complete nonlinear model) and the value of certain system parameters such as the rope length and the payload mass may significantly vary during the operations. Moreover, in the minimisation of the motion time of the payload the trolley driving motor constraints have to be taken into account.

Different open-loop and closed-loop control solutions have been proposed in the literature that address the problem with regard to the open-loop (i.e. the motion planning) approaches, in Sakawa and Shindo [1] an optimal motion law that minimises the swing of the load is devised. In Auernig and Troger [2] a time optimal control law is derived by using an extension of the Pontryagin maximum principle. Then, in Starr [3] and Strip [4] a family of acceleration strategies is selected in order to achieve a swing-free movement and in Singhose and Towell [5] and Singhose *et al.* [6] the input shaping technique is adopted to reduce the residual vibration.

Alternatively, with regard to the closed-loop approaches, in Moustafa [7] and Moustafa and Ebeid [8] a state-feedback strategy on the linearised system is implemented, whilst in Butler *et al.* [9] a model reference adaptive control is used. In Yoshida and Kawabe [10] and in Burg *et al.* [11] a saturating control law is proposed and in Cheng and Chen [12] a controller, which combines a feedback linearisation approach and a time delay control scheme, is chosen. In Gao and Chen [13, 14] and Levine [15] a flatness-based approach [16] is exploited to derive a feedforward/feedback control strategy. In Corriga *et al.* [17] the state-feedback control law results in an implicit gain scheduling controller in order to cope with the time varying parameter that represents the rope length. A variable structure controller has also been applied in Er *et al.* [18]. Finally, different solutions involving fuzzy logic and neural networks have also been proposed [19–23].

However, despite the wide range of proposed approaches weaknesses still remain. In many cases system uncertainties and actuator constraints are not taken into account and often the controller design is done by optimising only one side of the problem, i.e. either the travelling time of the payload is minimised or the swinging effect is reduced. Therefore, it seems that a design technique that is capable of achieving different goals in a unified framework is still lacking.

We propose a methodology, based on dynamic inversion, for the design of a robust feedforward/feedback control scheme which allows us to significantly reduce the transient sway and residual oscillation of the payload, whilst minimising the travelling time and taking into account the trolley's actuator constraints. The methodology basically consists of linearising the system and dampening it by means of a robust state-feedback controller. Then, after

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having defined a family of desired smooth polynomial motion laws for the payload (whose properties in the context of dynamic inversion have been discussed in Piazzi and Visioli [24]), depending on the motion time, the corresponding family of reference inputs is subsequently determined by inverting the nominal closed-loop system. Finally, with a worst-case approach, the motion time can be opportunely minimised ensuring that limits on the trolley's actuator are not exceeded.

In the literature, the use of dynamic inversion procedures has been proposed for a variety of output tracking problems; for example, see Piazzi and Visioli [25] and references therein contained for the case of nonlinear and linear systems and Piazzi and Visioli [26, 27] for applications to mechanical systems.

2 Model description and problem formulation

An overhead crane can be sketched as in Fig. 1, where m_c is the trolley mass, l is the rope length, m_L is the hook/payload mass, x_1 is the trolley position, x_3 is the rope angle and u is the force applied to the trolley. Some assumptions can be made to simplify the model [28]:

(a) the dynamics and nonlinearities of the driving motor are neglected;
(b) the trolley moves along the track without friction or slip;
(c) the rope has no mass and elasticity;
(d) there is no damping of the pendulum;
(e) the load is regarded as a point mass.

It has to be noted that assumption (a) is justified if the controller design assures that $|u|$ and $|\dot{u}|$ are not excessively large. Hence, the system can be modelled by two nonlinear second order differential equations (g is the gravity acceleration):

$$\begin{aligned}(m_L + m_c)\ddot{x}_1 + m_L l(\ddot{x}_3 \cos x_3 - \dot{x}_3^2 \sin x_3) &= u \\ m_L \ddot{x}_1 \cos x_3 + m_L l \ddot{x}_3 &= -m_L g \sin x_3\end{aligned} \quad (1)$$

which, solving for $\ddot{x}_1$ and $\ddot{x}_3$, can be rewritten as:

$$\ddot{x}_1 = -\frac{m_L \cos x_3 \sin x_3 g + m_L l \dot{x}_3^2 \sin x_3 + u}{-m_L - m_c + m_L \cos^2 x_3} \quad (2)$$

$$\ddot{x}_3 = \frac{\cos x_3 m_L l \dot{x}_3^2 \sin x_3 + u \cos x_3 + m_L g \sin x_3 + m_c g \sin x_3}{l(-m_L - m_c + m_L \cos^2 x_3)} \quad (3)$$

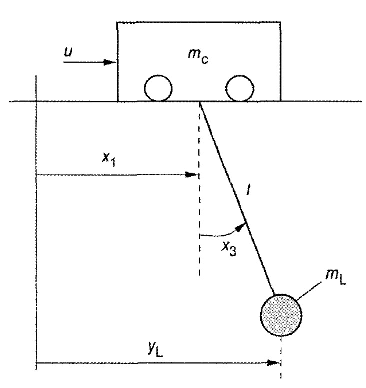

Fig. 1 *Sketch of an overhead crane*

The nonlinear model can be expressed in a state space form by defining $x_2 = \dot{x}_1$ and $x_4 = \dot{x}_3$. Moreover, assuming small deflection angles x_3's and small angular velocities x_4's (it will be shown in the following Section that these hypotheses are satisfied by the designed controller), the dynamic model can be linearised by simply imposing $\cos x_3 \approx 1$, $\sin x_3 \approx x_3$, $\sin^2 x_3 \approx 0$ and $x_4^2 \approx 0$. Hence, the following linear state space model results:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u \quad (4)$$

where $\boldsymbol{x} = [x_1\ x_2\ x_3\ x_4]^T$ and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{m_L}{m_c} g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{(m_L + m_c)g}{m_c l} & 0 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ \frac{1}{m_c} \\ 0 \\ -\frac{1}{m_c l} \end{bmatrix} \quad (5)$$

It has to be noted that the eigenvalues of the system are:

$$s_{1,2} = 0$$

and

$$s_{3,4} = \pm j\sqrt{1 + m_L/m_c}\sqrt{g/l}$$

that is, a double pole in the origin of the s-plane and a couple of pure imaginary poles. The hook/payload position is defined as:

$$y_L = x_1 + l \sin x_3 \cong x_1 + l x_3$$

or, alternatively

$$y_L \cong \boldsymbol{c}\boldsymbol{x} \quad \boldsymbol{c} = [1\ \ 0\ \ 1\ \ 0]$$

Then, the resulting transfer function between the force applied to the trolley and the payload position is:

$$G_{uy_L}(s) = \frac{g}{s^2[m_c l s^2 + (m_L + m_c)g]} \quad (6)$$

The addressed problem is to design a feedforward/feedback strategy in order to obtain a minimum-time movement of the payload starting from a given initial position y_{L_i} to a final one y_{L_f} subject to: (i) constraints on the force exerted by the actuator on the trolley, for example the values of $|u|$ and $\dot{u}$ have to be limited; and (ii) the transient sway and the residual oscillation of the payload have to be reduced as much as possible. Without loss of generality in the following we will assume $y_{L_i} = 0$. Moreover, it has to be taken into account that in practical cases the values of the mass of the hook/payload and of the rope length are uncertain, although it is known that they belong to a given interval, that is $m_L \in [m_L^-, m_L^+]$ and $l \in [l^-, l^+]$. Finally, the rope length can vary during the motion, in cases where the payload is hoisted or lowered.

Remark 1: Note that the variation of the rope length during the motion is not modelled in (4) and (5) and it will therefore be treated as an unmodelled uncertainty in the proposed design method.

3 Dynamic-inversion-based control design

Basically, the control design methodology consists of first determining a state-feedback control law whose aim is to assure the robust stability of the system and to damp it. In this phase it is necessary to augment the system (5) in order to assure that a null steady-state error is achieved at the end of the motion in spite of possible external disturbances (for example friction effects). Then, after having defined a suitable parameterised family of desired (output) function which depend on the motion time, the corresponding family of command input function is derived by inverting the closed-loop system. The motion time can then be minimised taking into account actuator constraints and the parameter uncertainties. The different design phases are described in the following Sections.

3.1 State-feedback control design

In order to assure a null steady-state error for the case of external disturbances, it is necessary to insert an integral part in the controller. Hence, we define a new state x_5 such that:

$$\dot{x}_5 = x_1 - r$$

where r is the command input function to be determined by means of a dynamic inversion. Thus, we can define the state-feedback control law as:

$$u = \boldsymbol{k}_e^T \boldsymbol{x}_e \tag{7}$$

where $\boldsymbol{k}_e = [k_1 \ \cdots \ k_5]^T$ and $\boldsymbol{x}_e = [x_1 \ \cdots \ x_5]$.

Different algorithms can be employed for this purpose (see for example Ackermann [28], Jaulin and Walter [29] and Piazzi and Marro [30]), assuring also that the real part of the characteristic polynomial roots is less than an arbitrarily chosen real number $\rho < 0$. It is important to stress that the resulting robust pole placement is not a critical issue for the performance of the overall control system, due to the adopted smooth dynamic inversion concept, this point will be illustrated in the following. Moreover, it also has to be highlighted that the trolley position and the rope angle can be measured by appropriate sensors and consequently the other state vector elements can be reconstructed [17, 31]. Alternatively, an estimation $\hat{\boldsymbol{x}}$ of the state vector $\boldsymbol{x}$ (obviously there is no need to estimate the state x_5) can be obtained by solely measuring the trolley position, by means of a simple Luenberger observer, based on the nominal values of the system parameters, again without impairing the control performances, this will be shown in the simulation examples in Section 4. Formally, the observer can be described by the following expression:

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{b}u + \boldsymbol{h}(x_1 - \boldsymbol{c}\boldsymbol{x}) \tag{8}$$

where $\boldsymbol{h}$ is a suitably chosen vector such as the poles of the observer are sufficiently far in the left-half plane with respect to the poles of the closed-loop system. Therefore, it follows that the resulting control law is actually:

$$u = \boldsymbol{k}^T \hat{\boldsymbol{x}} + k_5 x_5 \quad \boldsymbol{k}^T = [k_1 \cdots k_4]^T \tag{9}$$

The overall control scheme is depicted in Fig. 2. Once the state-feedback controller has been determined, the transfer function of the closed-loop linearised system between the

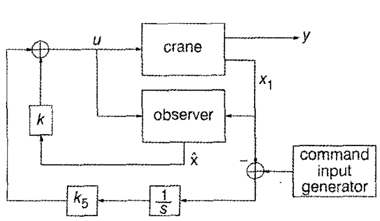

Fig. 2 *The overall control scheme*

command input r and the hook/payload position y_L becomes:

$$G_{ry_L}(s) = \frac{k_5 g}{m_c l s^5 + (k_4 - k_2 l)s^4 + (m_c g - k_1 l + k_3 + m_L g)s^3 + (k_5 l - k_2 g)s^2 - k_1 g s + k_5 g} \tag{10}$$

3.2 Output function design

In order to find a suitable command input function by means of a dynamic inversion, it is first necessary to define a desired motion law for the payload. In this context, an appropriate choice is to adopt a 'transition' polynomial which depend on the motion time τ and which ensures that the output function derivatives until the νth order are null for $t = 0$ and $t = \tau$. Its expression can be written as [24]:

$$y(t;\tau) = y_{L_f} \frac{(2\nu+1)!}{\nu!\tau^{2\nu+1}} \sum_{i=0}^{\nu} \frac{(-1)^{\nu-i}}{i!(\nu-i)!(2\nu-i+1)} \times \tau^i t^{2\nu-i+1} \quad t \in [0, \tau] \tag{11}$$

Outside the interval $[0, \tau]$, $y(t; \tau)$ is simply defined as $y(t; \tau) = 0$ if $t \leq 0$ and $y(t; \tau) = y_{L_f}$ if $t \geq \tau$. An alternative expression of (11) is given by the integral representation:

$$y(t;\tau) = y_{L_f} \frac{(2\nu+1)!}{(\nu!)^2 \tau^{2\nu+1}} \int_0^t \xi^\nu (\tau - \xi)^\nu d\xi \quad t \in [0, \tau] \tag{12}$$

that makes clear that $y(t; \tau)$ is monotonically increasing since $\dot{y}(t; \tau)$ is positive over $(0, \tau)$. As a consequence, the planned motion of the end-effector is, by construction, free of oscillatory modes.

In (11) the integer ν (i.e. the order of the polynomial) can be chosen according to the following property [24], taking into account the dynamic inversion procedure to be subsequently performed in order to determine the command input function $r(t; \tau)$ (as usual $C^{(\nu)}$ denotes the space of real functions that have continuous derivatives till to the νth order).

Property 1: Consider the system described by the transfer function (10). If $y(t; \tau) \in C^{(\nu)}$, then it is functional reproducible by a unique $r(t; \tau) \in C^{(\nu - 5)}$.

In other words, the order of the output function can be selected in order to ensure the continuity of the input function until a predefined order.

Remark 2: The choice of a polynomial function with respect to other functions (see e.g. Perez and Devasia [32]) is justified by the fact that in this way it is possible to obtain a simple analytic expression of the corresponding input function (see the following Section) and therefore the overall computational procedure is greatly simplified.

3.3 Command input function design via dynamic inversion

Once the non-oscillatory desired output function $y_L(t; \tau)$ has been selected, the corresponding command input function $r(t; \tau)$ can be determined by means of a dynamic inversion procedure on the nominal linearised closed-loop system. Consider the transfer function (10) corresponding to the nominal parameters $m_L^0 = (m_L^- + m_L^+)/2$, $l^0 = (l^- + l^+)/2$ and denote by $R(s; \tau)$ and $Y_L(s; \tau)$ the Laplace transforms of $r(t; \tau)$ and $y_L(t; \tau)$ respectively. Considering that both $r(t; \tau) = 0$ and $y_L(t; \tau) = 0$ for all $t<0$ (at $t=0^-$ all initial conditions are zeros), it follows that:

$$Y_L(s;\tau) = G_{ry_L}(s; m_L^0, l^0)\,R(s;\tau)$$

Then, the required dynamic inversion on the nominal closed-loop linearised system can be simply performed by computing the inverse of $G_{ry_L}(s; m_L^0, l^0)^{-1}$:

$$R(s;\tau) = G_{ry_L}(s; m_L^0, l^0)^{-1}\,Y_L(s;\tau)$$

Hence, applying the inverse Laplace transform on $R(s;\tau)$ we easily obtain:

$$\begin{aligned} r(t;\tau) &= \frac{m_c l^0}{k_5 g} y_L^{(5)}(t;\tau) + \frac{k_4 - k_2 l^0}{k_5 g} y_L^{(4)}(t;\tau) \\ &+ \left(\frac{m_L^0 + m_c}{k_5} + \frac{k_3 - l^0 k_1}{k_5 g}\right) y_L^{(3)}(t;\tau) + \left(\frac{l}{g} - \frac{k_2}{k_5}\right) \\ &\times y_L^{(2)}(t;\tau) - \frac{k_1}{k_5} y_L^{(1)}(t;\tau) + y_L(t;\tau) \end{aligned} \quad (13)$$

Remark 3: It is worth stressing that (13) is simply formed by an algebraic linear combination of $y_L(t;\tau)$ and its derivatives (till to the fifth order) without any inclusion of zero dynamic modes, due to the absence of finite zeros in $G_{ry_L}(s; m_L^0, l^0)$. Moreover, the final position parameter y_{L_f} appears just as a scaling factor in the expression of $r(t;\tau)$ so that the reference function is easy to calculate once the travelling time τ is known.

3.4 Minimisation of the motion time

Once the family of command input functions $r(t;\tau)$ has been determined by means of (13), the motion time τ can be minimised taking into account the crane dynamics and the trolley's actuator constraints. Specifically, choosing a worst-case approach, the absolute values of the manipulative input u and its derivatives (till a prefixed order) have to be limited for any possible values of the uncertain parameters m_L and l. Formally, the following semi-infinite optimisation problem has to be solved:

$$\min \tau \quad (14)$$

subject to ($i=0,\ldots,n$):

$$|u^{(i)}(t;\tau,m_L,l)| \le u_{\max}^{(i)} \quad \forall t \in [0,\tau] \quad \forall m_L \in [m_L^-, m_L^+] \quad \text{and} \quad \forall l \in [l^-, l^+]$$

where $u_{\max}^{(i)}$, $i=0,\ldots,n$ are fixed in such a way that they correspond to physical limits of the trolley's actuator; note that n have to be chosen according to $0 \le n \le v-5$ provided that $y(t;\tau) \in C^{(v)}$.

The derivatives of the control function $u(t;\tau,m_L,l)$ are to be evaluated considering the exact nonlinear crane model as described in (1). The optimal solution τ^* of the above problem can be found by means of the following typical bisection algorithm:

Step 1: Set $\tau_{\min}=0$.
Step 2: Determine an initial value for $\tau_{\max}$ such as $\max_{t\in[0,\tau]}|u^{(i)}(t;\tau_{\max},m_L,l)| \le u_{\max}^{(i)}$, $i=1,\ldots,n$, $\forall m_L \in [m_L^-, m_L^+]$ and $\forall l \in [l^-, l^+]$.
Step 3: Set $\tau=(\tau_{\min}+\tau_{\max})/2$.
Step 4: If $\max_{t\in[0,\tau]}|u^{(i)}(t;\tau,m_L,l)| \le u_{\max}^{(i)}$, $i=1,\ldots,n$, $\forall m_L \in [m_L^-, m_L^+]$ and $\forall l \in [l^-, l^+]$ then set $\tau_{\max}=\tau$ else set $\tau_{\min}=\tau$.
Step 5: If $(\tau_{\max}-\tau_{\min})>\varepsilon$ then goto 3.
Step 6: Set $\tau^*=\tau_{\max}$.
Step 7: Find.

Note that the precision parameter ε, which determines the terminal condition of the algorithm at step 5, is arbitrarily fixed.

Remark 4: The initial value of $\tau_{\max}$ can be easily found starting from a reasonable value and if it does not satisfy the constraints of step 2, multiplying it repeatedly by a constant $\lambda>1$ until the condition is true.

Remark 5: In order to verify the conditions of step 4 (or step 2), the determination of the maximum

$$\max\{|u^{(i)}(t;\tau,m_L,l)| : t\in[0,\tau] \quad m_L \in [m_L^-, m_L^+] \; l \in [l^-, l^+]\}$$

is required. This is a difficult problem because, in general, $u^{(i)}(t;\tau,m_L,l)$ does not admit a closed-form expression in the arguments t, m_L and l. However, from a practical point of view, the problem can be reasonably relaxed by replacing the intervals $[m_L^-, m_L^+]$ and $[l^-, l^+]$ with finite discretisations.

Remark 6: Although the overall control system design requires us to perform several steps, its practical implementation should not be a problem in modern cranes, as already demonstrated in many works [9–11, 19, 33] where complex controllers have been successfully tested on real systems.

4 Simulation results

As an illustrative example we considered a crane with trolley mass $m_c=1000$ kg. The nominal payload mass value is $m_L^0=1500$ kg and the rope nominal length is $l^0=8$ m. The uncertainty on these two values is $\pm 50\%$ for the payload mass, i.e. $m_L^-=750$ kg, $m_L^+=2250$ kg, and $\pm 25\%$ for the rope length, i.e. $l^-=6$ m and $l^+=10$ m. The state-feedback vector has been calculated in such a way that the poles location in the s-plane for the nominal closed-loop system is $[-1.5\ -1.2\ -0.9\ -0.8\ -0.6]$. This results $\boldsymbol{k}=[-4386.1\ -8649.5\ 29\,138.3\ -36\,495.2\ -793.5]$. Closed-loop pole locations for limiting values of the system parameters are reported in Table 1. In the simulation results, the state vector $\boldsymbol{x}$ has been reconstructed by means of a classic Luenberger observer, based on the nominal linearised system, using only the measure of the trolley position. The pole locations of the observer system are $[-10\ -15\ -20\ -25]$.

We considered the following payload motion function $y_L(t;\tau)\in C^{(5)}$, so that $r(t;\tau)\in C^{(0)}$ and $u(t;\tau,m_L,l)\in C^{(1)}$ ($v=5$):

$$y_L(t;\tau) = y_{L_f}\left(-\frac{252}{\tau^{11}}t^{11} + \frac{1386}{\tau^{10}}t^{10} - \frac{3080}{\tau^9}t^9 + \frac{3465}{\tau^8}t^8 - \frac{1980}{\tau^7}t^7 + \frac{462}{\tau^6}t^6\right), \quad t\in[0,\tau]$$

The absolute value of the maximum force $|u|$ that can be applied to the trolley is 2000 N, whilst the maximum value of its derivative is 1600 N/s. The payload has to be moved from zero to 10 m.

Table 1: Poles location for different values of system parameters

System parameter values	Closed-loop poles of linearised system
$m_L=m_L^-$, $l=l^-$	$-1.08\pm 2.24i$, -0.81, $-0.41\pm 0.20i$
$m_L=m_L^-$, $l=l^+$	$-0.38\pm 0.17i$, -4.18, $-0.39\pm 0.84i$
$m_L=m_L^+$, $l=l^-$	$-1.30\pm 2.67i$, -0.37, $-0.40\pm 0.39i$
$m_L=m_L^+$, $l=l^+$	$-0.40\pm 0.60i$, -3.73, -0.81, -0.40

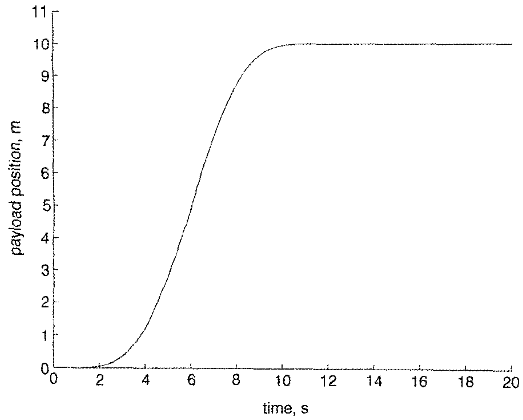

Fig. 3 *The desired payload motion law*

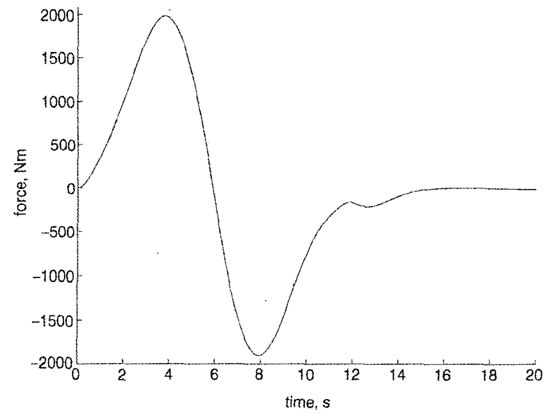

Fig. 5 *Force applied to the trolley when* $m_L = m_L^+$ *and* $l = l^+$

The optimisation procedure presented in Section 3.4 has been applied with $\varepsilon = 0.1$ taking into account limits on the force to be exerted on the trolley and its first derivative ($n = 1$). The minimum motion time is $\tau^* = 12.0$ s.

Note that all the results shown in the following have been obtained by means of simulations performed taking into account the full nonlinearities of the crane dynamics and the payload is hoisted during the motion with a constant velocity equal to 0.1 m/s, starting at time $t = 0$ and ending at time $t = \tau$. Specifically, the rope length is time-varying according to $l = l_0 + \alpha t$ with $\alpha = 0.1$ m/s, $t \in [0, \tau]$ and l_0 (the initial rope length) can span in the interval $[l^-, l^+]$.

In Fig. 3 the nominal payload motion law is reported, whilst the optimal command input $r(t;\ \tau^*)$, obtained by inverting the nominal system is shown in Fig. 4. Then, in Fig. 5 we show $u(t;\ \tau^*, m_L^+, l^+)$, that is the case in which the force limit is attained. It appears that the capacities of the actuator are fully exploited.

In order to verify the performances of the devised overall controller, several simulations have been performed, with different values of the payload mass and the initial rope length, over their range of uncertainty. With a tight gridding over the uncertain parameter domain, the maximum overshoot of the payload has been plotted in Fig. 6. Moreover, Fig. 7 is a plot of the following index:

$$J := \max_{t \geq \tau^*} \frac{|y_L(t) - y_{L_f}|}{y_{L_f}} \times 100 \tag{15}$$

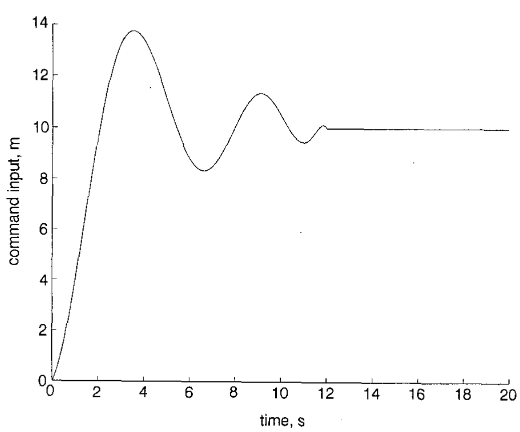

Fig. 4 *The optimal command input function*

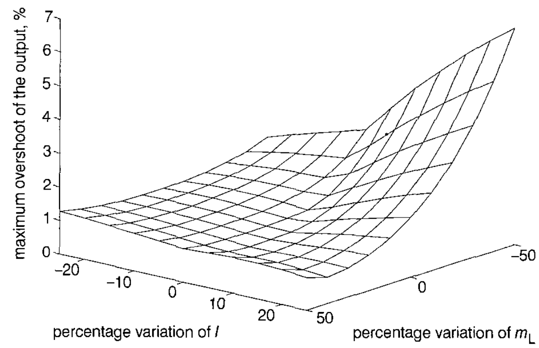

Fig. 6 *Percentage overshoot of the payload with different values of the system parameters*

which might be considered as a better measure of the residual amplitude, since in any case it somehow penalises a slow motion even if it does not present an overshoot. In this context, the worst-case payload motion, which occurs when $m_L = m_L^-$ and $l = l^+$ is shown in Fig. 8. The resulting maximum overshoot is 6.4% and the same value results for J. The corresponding rope angle, which assures that the system linearisation is effective, is plotted in Fig. 9.

From these results it is clear why that the control strategy is effective since it also assures a small swing effect in presence of large uncertainties, without impairing the efficiency of the operations. In addition, the transient sway and residual oscillation are eliminated in practice

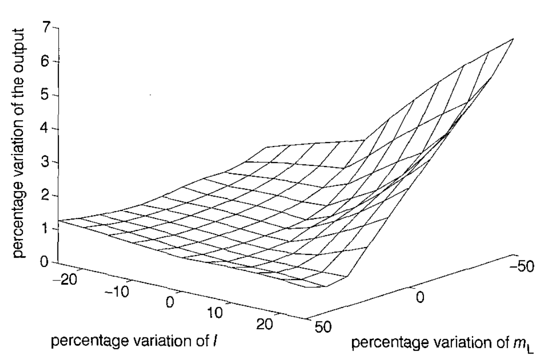

Fig. 7 *J index with different values of the system parameters*

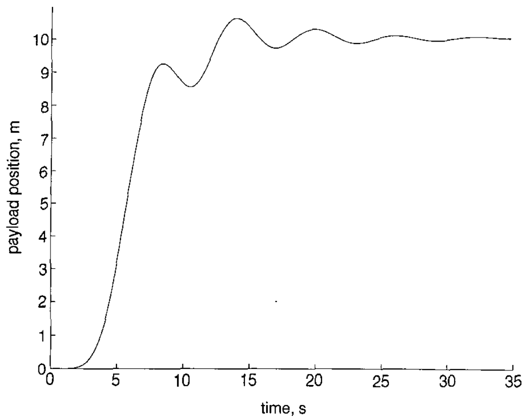

Fig. 8 *Worst-case payload motion (for $m_L = m_L^-$ and $l = l^+$)*

when the mass of the payload is known, which is a highly desirable feature because sometimes this information is available (for example when no payload is carried and the only mass is the one of the hook).

Finally, we also evaluated the performances of the system in the presence of friction effects. Namely, we considered the following friction force, proportional to the trolley velocity, that occurs on the trolley itself:

$$u_f = k_f x_2 \quad k_f = 200$$

The output in the case $m_L = m_L^-$ and $l = l^+$ (note that again the payload is hoisted during the motion) is almost identical to the one shown in Fig. 8. It turns out that no significant differences emerge with respect to the case in which the friction is not present and therefore to a certain degree the methodology also appears to be robust to unmodelled dynamics.

Remark 7: With respect to the solutions proposed in Starr [3] and Strip [4], the technique proposed in this paper is simpler, as it does not require a major effort in designing the controller for accurately tracking the determined path of the trolley. This might be difficult especially when a heavy payload is carried, as its motion represents a significant disturbance for the trolley motion control system. Actually, the dynamic-inversion-based method can be thought of as an alternative to the use of input shaping [5, 6]. Note that a thorough comparison between the two approaches for the reduction of the residual vibration in servosystems endowed with elastic transmissions is reported in Piazzi and Visioli [26].

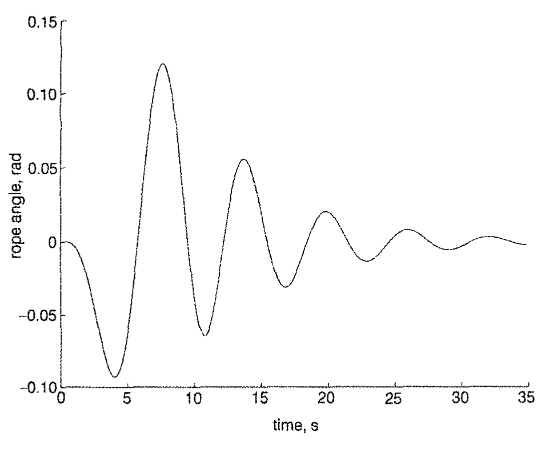

Fig. 9 *Rope angle for the worst-case payload motion*

5 Conclusions

A methodology for the design of a feedforward/feedback control strategy for an overhead crane has been proposed. Despite its simplicity, it is based on a linearisation of the model and it exploits basic control concepts, it is significantly robust to system uncertainties and at the same time it allows us to minimise the travelling time taking into account actuator constraints. Moreover, it provides a useful design flexibility, since the worst-case overshoot can be reduced by increasing the travelling time. Results based on the nonlinear model of the crane and in which the hoisting of the load and the presence of friction effects have been considered, and they show the effectiveness of the approach.

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