

m/s<sup>2</sup>, and the absorbed power is an incredibly low 0.35 W (a 95% reduction). Figure 15 is a similar comparison for 40.2 kph (25 mph). The passive response (top) exhibits peak accelerations in excess of 15.0 m/s<sup>2</sup>, and absorbed power of 13.85 W, more than twice the limiting threshold [25]. Peak accelerations for the active suspension are below 5.0 m/s<sup>2</sup>, and the absorbed power is a remarkably low 1.49 W (an 89% reduction).

## VIII Conclusions and Recommendations

This paper details the development and real-time implementation of intelligent parameter estimation and Intelligent Feedback Linearization (IFL) to improve the ride quality and handling performance of off-road active suspension systems. The IFL controller combines Radial Basis Function Networks (RBFNs) with an adaptive control strategy to cancel undesired nonlinearities, thus facilitating the use of linear control laws. Experimental results from a quarter-car test rig demonstrate 60% improvements in ride quality relative to a baseline (non-adapting) controllers. Additional, field trial results from a HMMWV implementation clearly demonstrate the performance of quarter-vehicle parameter estimation and control algorithms for improved ride quality. 95% reductions in absorbed power and 65% reductions in peak sprung mass acceleration have been documented using this IFL approach.

The IFL controller design followed a logical progression from concept and computer simulation [26], to real-time experiment, to full-vehicle implementation. Testing and refinement of the full-vehicle controller, currently installed on a HMMWV, is still underway. Current research at UT-CEM is addressing full-vehicle (dynamically coupled) control algorithms and bump-stop avoidance algorithms, as the energy transmitted to occupants relies heavily on avoiding the shocks associated with travel space exhaustion.

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## Point-to-Point Motion Planning for Servosystems With Elastic Transmission Via Optimal Dynamic Inversion<sup>1</sup>

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*A new technique, based on dynamic inversion, for the residual vibration reduction in the point-to-point motion of servosystems with elastic transmission is presented. The methodology consists of defining a suitable motion law for the load, and subsequently determining, via dynamic inversion, the corresponding command function for the system. The method inherently assures the robustness of the control scheme despite inaccuracies in the estimation of the stiffness constant and of the damping of the transmission. The main contribution of the paper lies in the definition of a simple optimization procedure which allows the system inversion point that minimizes the residual vibration to be found. Experimental results show that in this way the identification phase is less critical and performances can be significantly improved.*  
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**Keywords:** Oscillatory Systems, Open-Loop Control, Dynamic Inversion, Optimization

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# 1 Introduction

High performances servosystems might suffer from the presence of elasticities in the transmission, which produces a residual vibration at the end of a point-to-point motion. This fact introduces limitations in the reduction of the working cycle time, since the oscillation has to vanish before achieving the desired accuracy of the positioning system. This problem has been addressed by a number of researchers in the past and two strategies have been followed: either implementing a closed-loop control scheme, in which the state of the dynamic system has to be known during the motion (and therefore apposite sensors have to be adopted) or an open-loop control scheme which relies on an appropriate motion planning methodology. In the context of the open-loop strategy, the input shaping technique has been successfully developed over the last decade (see e.g. [1,2]). Basically, it consists of convolving a sequence of impulses, also known as the input shaper, with a desired system command to generate the system command that is then used to drive the system.

In a previous paper [3] we proposed an alternative approach to the input shaping technique. It is based on the concept of dynamic inversion and it allows minimization of the motion time while taking into account actuator constraints. The general idea is to first define an arbitrarily smooth closed-form polynomial motion function, parameterized by the time interval  $\tau$ , for the load of the system to avoid oscillations during and at the end of the motion [4]. Then, by means of dynamic inversion the actual command input that causes the desired planned load motion is derived. Simulation and experimental results have proven the effectiveness of the method, with respect to the input shaping technique, and how it is inherently robust to modeling errors.

In this paper, we further develop the system-inversion-based methodology. Rather than considering a priori the uncertainties of the model, we perform a simple procedure to search for the optimal inversion point to minimize the residual vibration amplitude. In other words, starting from values of the system parameters determined by means of a simple identification experiment, we modify them in the system model adopted for the dynamic inversion, through repetitive experiments, until the residual vibration amplitude is minimized.

## 2 Dynamic Inversion Based Motion Planning

The presence of a gearbox in a mechanical positioning servo-system generally introduces an elastic element which can be simply described by the model shown in Fig. 1 where  $y$  is the coordinate representing the motor shaft displacement,  $x$  is the coordinate representing the mass displacement,  $m$  is the load mass,  $k$  the stiffness constant, and  $c$  the damping of the transmission [5]. The well-known linear relation between  $x$  and  $y$  has the following differential form:

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky \tag{1}$$

which can be rewritten as:

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n^2z = -\ddot{y}$$

where  $z = x - y$ ,  $\omega_n = \sqrt{k/m}$  rad·s<sup>-1</sup> is the frequency of the oscillatory mode and  $\xi = c/2m\omega_n$  is the damping ratio.

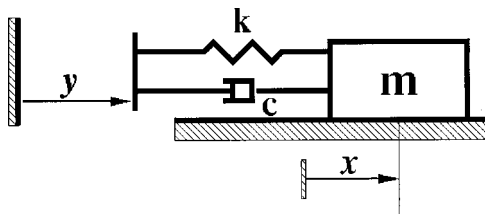


Fig. 1 Model of an elastic transmission

In general (we refer for example to the motion planning of industrial robot manipulators), the elasticity of the transmission is not taken into account and the motion is planned on  $y$  (the input of the system (1)), exploiting the full capabilities of the actuator and considering the mass  $m$  rigidly linked to the motor. When, however, excessive vibrations occur, the velocities, accelerations and jerks of the trajectory have to be reduced by increasing the motion time and therefore limiting the performances of the system. This drawback can be overcome by adopting a system-inversion-based methodology which will be briefly described in the following. For details see [3].

Define a point-to-point motion law from position 0 to  $q$  for the load in the time interval  $[0, \tau]$  using the “transition” polynomials introduced in [4]:

$$x(t; \tau) = q \frac{(2h+1)!}{h! \tau^{2h+1}} \sum_{i=0}^h \frac{(-1)^{h-i}}{i!(h-i)!(2h-i+1)} t^i \tau^{2h-i+1}. \tag{2}$$

Outside the interval  $[0, \tau]$ ,  $x(t; \tau)$  is simply defined as  $x(t; \tau) = 0$  if  $t \leq 0$  and  $x(t; \tau) = q$  if  $t \geq \tau$ . In (2) the integer  $h$  can be arbitrarily chosen in order to assure  $x(t; \tau) \in C^{(h)}$  over  $\mathbb{R}$ , i.e.,  $x(t; \tau)$  has continuous derivatives up to the  $h$ th order. Note that  $x(t; \tau)$  is monotonically increasing and as a consequence, the planned motion of the mass  $m$  is, by construction, free of oscillatory modes.

Consider the transfer function of the system (1):

$$G(s) = \frac{X(s)}{Y(s)} = \frac{cs + k}{ms^2 + cs + k} \tag{3}$$

Applying the Laplace transform operator  $\mathcal{L}$  both to  $y(t; \tau)$  and  $x(t; \tau)$ , the closed-form expression of the parameterized input function for  $t \geq 0$  (obviously  $y(t; \tau) = 0$  if  $t < 0$ ) which causes the desired output function can be calculated as:

$$y(t; \tau) = \mathcal{L}^{-1}[G^{-1}(s)X(s; \tau)] = \frac{m}{c} \dot{x}(t; \tau) + \left(1 - \frac{mk}{c^2}\right) x(t; \tau) + \frac{mk^2}{c^3} e^{-(k/c)t} \int_0^t e^{(k/c)v} x(v; \tau) dv. \tag{4}$$

Note that  $y(t; \tau)$  is all over bounded because the excited zero mode  $e^{-(k/c)t}$  is stable. In order to obtain  $y(t; \tau)$  belonging to  $C^{(l)}$  it is necessary that, by virtue of (4),  $x(t; \tau)$  belong to  $C^{(l+1)}$ , i.e.,  $h = l + 1$ . In particular, to at least ensure the continuity of the velocity input function, the constraint  $h \geq 2$  must be satisfied.

At this point the optimization procedure described in [3] for the minimization of the motion time subject to actuator constraints can be readily applied.

## 3 Dynamic Inversion Point Optimization

In the previous section it has been exposed a dynamic inversion based synthesis of the motion input that depends, besides  $\tau$ , on the parameters  $m$ ,  $c$ , and  $k$ . In particular, the linkage parameters  $c$  and  $k$ , which are not exactly known in many practical cases, can be explicitly indicated as formal parameter arguments in  $y(t; c, k, \tau)$ . Thus, the problem of the optimal selection of  $c \geq \delta_c$  and  $k \geq \delta_k$  in  $y(t; c, k, \tau)$  arises, where  $\delta_k$  and  $\delta_c$  are given arbitrarily small positive values of the natural frequency and of the damping ratio. We propose to choose  $c$  and  $k$  in order to minimize the amplitude of the actual residual vibration (the transient motion of the load for  $t \geq \tau$ ). Application of input  $y(t; c, k, \tau)$  to the actual servo determines an output motion denoted as  $x(t; c, k, \tau)$ . Hence, the residual vibration amplitude can be defined as:

$$J = J(c, k) := \max_{t \geq \tau} |x(t; c, k, \tau) - q|. \tag{5}$$

The addressed motion planning problem is therefore posed as the following minimization problem:

$$\min_{c \geq \delta_c, k \geq \delta_k} J(c, k). \quad (6)$$

Searching for a global solution of the optimization problem is extremely difficult because it has to be solved by means of suitably arranged experimental trials. Therefore, we propose a practicable local optimization procedure based on a simplified coordinate descent method [[6], p. 227] which is in any case capable to significantly improve the control system performances. Hence, the minimizers  $c^*$  and  $k^*$  of problem (6) can be found by means of the following optimal inversion point (OIP) procedure.

OIP Procedure

- 1 Perform an identification experiment and estimate  $k_0$  and  $c_0$  (initial values).
- 2 Set  $k^* = k_0$ ,  $c^* = c_0$ , flag=0.
- 3 Perform an experiment and set  $I := J(c^*, k^*)$  and  $I^+ = I$ .
- 4 Set  $k^* = (1 + \varepsilon)k^*$ .
- 5 Perform an experiment and set  $I := J(c^*, k^*)$ .
- 6 If  $I < I^+$  then set flag=1 and go to 3.
- 7 If flag=1 go to 11.
- 8 Set  $k^* = (1 - \varepsilon)k^*$ . If  $k^* < \delta_k$  then set  $k^* = \delta_k$ .
- 9 Perform an experiment and set  $I := J(c^*, k^*)$ .
- 10 If  $I < I^+$  then go to 8.
- 11 Set flag=0.
- 12 Set  $c^* = (1 + \varepsilon)c^*$ .
- 13 Perform an experiment and set  $I := J(c^*, k^*)$ .
- 14 If  $I < I^+$  then set flag=1 and go to 12.
- 15 If flag=1 go to 19.
- 16 Set  $c^* = (1 - \varepsilon)c^*$ . If  $c^* < \delta_c$  then set  $c^* = \delta_c$ .
- 17 Perform an experiment and set  $I := J(c^*, k^*)$ .
- 18 If  $I < I^+$  then go to 16.
- 19 End.

The typical identification experiment that can be performed at step 1 might consist of applying a torque impulse to the motor and analyzing the oscillatory response of the load. From the evaluation of the frequency and of the decay ratio of the response, the stiffness constant and the damping ratio can be straightforwardly determined.

Parameter  $\varepsilon$  determines the velocity of the descent to the minimum and the precision in determining  $k^*$  and  $c^*$ . It is easy to adapt it in order to have a fast descent to the minimum at the beginning and then increasing the accuracy once we are closed to it (for example, on the practical grounds,  $\varepsilon$  can be initially fixed to 0.05).

*Remark 1.* It is very important to stress that the resulting optimal parameters do not necessarily coincide with the real values of the parameters of the physical system. In other words, minimizing the residual vibration does not mean in general that we have accurately identified the stiffness constant and the damping ratio of the system, since nonlinear effects, which are inevitably present in the system, are not included in the simple model (1).

#### 4 Experimental Setup and Results

The experimental setup consists of a testbed, depicted in Fig. 2, in which two carts, linked by a spring, slide on a stainless steel rectilinear guide (see Fig. 3 for a detail of the two carts). The first cart is connected to a belt which is moved through some pulleys by means of a brushless motor configured in torque mode, i.e., the signal given to the drive by the controller is a torque command. The overall reduction rate of the transmission system is known and therefore, we assume that the input reference function  $y(t)$  is the position of the first cart, rather than the position of the motor shaft.

Then, the position  $y(t)$  of the first cart is measured by means of an incremental encoder, mounted on the motor shaft, whose resolution is 4·1000 impulses per motor revolution. The second cart,

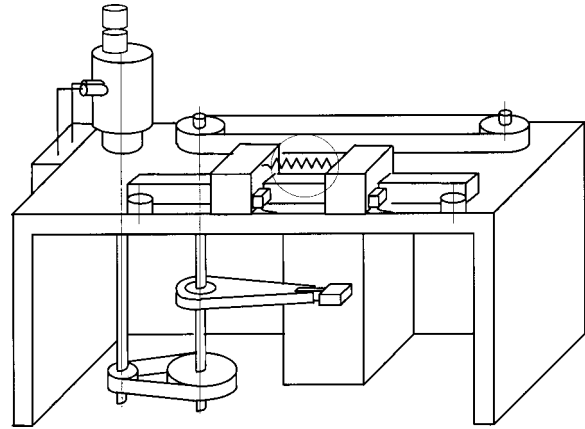


Fig. 2 Sketch of the experimental setup

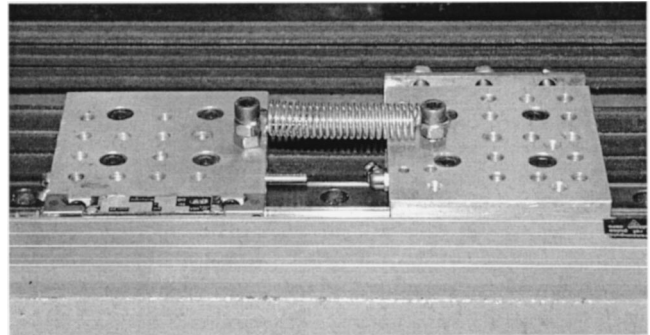


Fig. 3 The two carts linked with the spring adopted for the experiments

whose mass is 0.8 kg, is not actuated and its position  $x(t)$  is measured with an incremental linear scale with a resolution of 40  $\mu\text{m}$ .

The control system is implemented in a PC with I/O boards and the control frequency is 1 kHz. The position of the first cart is controlled by a standard Proportional-Integral-Derivative (PID) controller which has been accurately tuned by a trial and error procedure, in order to guarantee a very low positioning error during the motion.

A simple identification experiment, in which a torque impulse was applied to the first cart and the oscillatory response of the second cart has been analyzed, has been initially performed. The

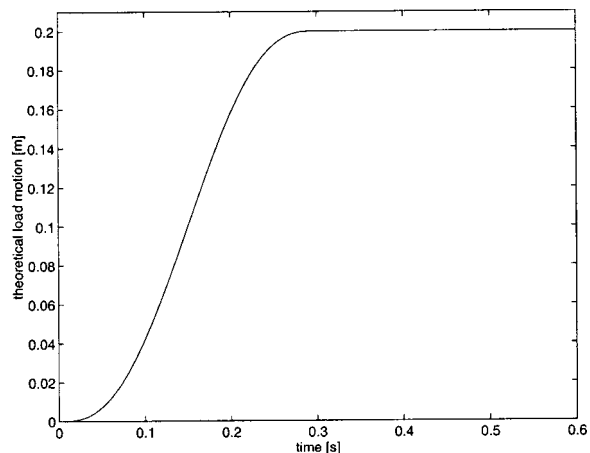
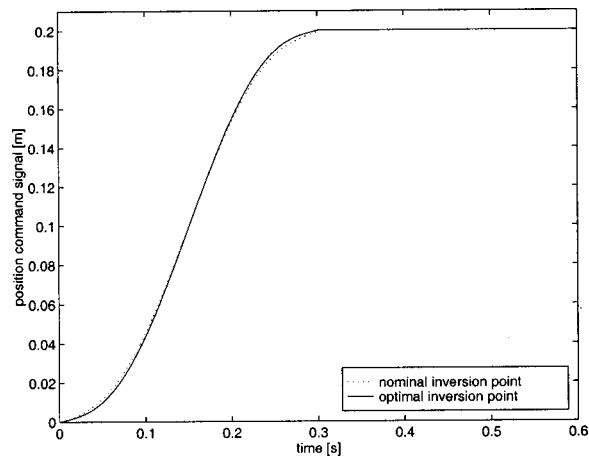
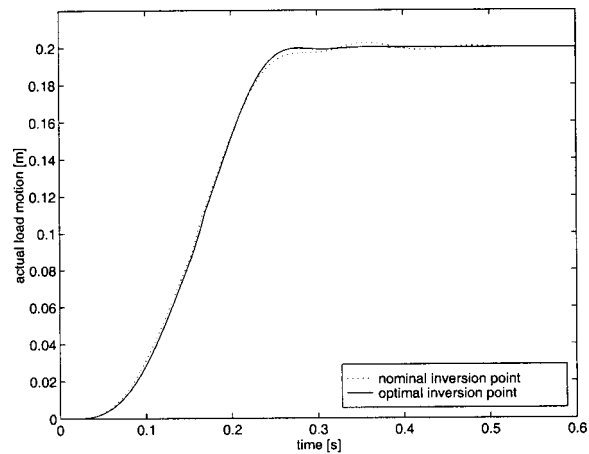


Fig. 4 The planned motion of the load



**Fig. 5 The command input function for the nominal and optimal dynamic inversion**



**Fig. 6 The actual load motion for the nominal and optimal dynamic inversion**

resulting values of  $k_0$  and  $c_0$  were  $1858.15 \text{ kg}\cdot\text{s}^{-2}$  and  $8.03 \text{ kg}\cdot\text{s}^{-1}$ , respectively, which corresponds to a nominal natural frequency of  $48.19 \text{ rad}\cdot\text{s}^{-1}$  and a nominal damping ratio of 0.10. Then, a load motion of  $q=0.2 \text{ m}$  to be accomplished in  $\tau=0.3 \text{ s}$  was planned (see Fig. 4). The value of the motion time have been selected in order to exploit the full dynamics of the actuator, without saturating. A polynomial output function of fifth order ( $h=2$ ) has been adopted, in order to guarantee the continuity of the input function until the first order, i.e., discontinuities in the acceleration reference signal are allowed. Thus, we have:

$$x(t) = 0.2 \left( \frac{6}{0.3^5} t^5 - \frac{15}{0.3^4} t^4 + \frac{10}{0.3^3} t^3 \right).$$

Then, the OIP procedure has been applied, having fixed  $\delta_c = \delta_k = 10^{-4}$ . At the end, it results in  $k^* = 3000$  and  $c^* = 0.001$ , that is, a natural frequency of  $61.24 \text{ rad}\cdot\text{s}^{-1}$  and a damping ratio of  $1.02 \cdot 10^{-5}$ . The resulting input function, obtained via dynamic inversion, is plotted in Fig. 5, where it is compared with the input function obtained by inverting the nominal system with  $k_0$  and  $c_0$ . The actual load motions for both the nominal and optimal dynamic inversion are plotted in Fig. 6, where it appears that significant improvement is achieved by using the OIP procedure. Note that the objective function  $J$  is reduced from  $2.7 \cdot 10^{-3} \text{ m}$  to  $0.6 \cdot 10^{-3} \text{ m}$ . Moreover, the steady-state value of  $0.2 \text{ m}$  is attained at  $t=0.4 \text{ s}$  for the optimal case and at  $t=0.54 \text{ s}$  for the nominal one.

## 5 Conclusions

In this paper we have presented an important development in the system-inversion-based technique for the reduction of the residual vibration in point-to-point motion of mechanical servosystems endowed with elastic transmissions. It has been shown that the use of the dynamic inversion methodology provides flexibility in the motion planning design, as it allows to easily cope with the actuator limits. Moreover, the identification phase can be kept at a very simple level, as the use of the polynomial functions ensure an inherent robustness to the system and the OIP procedure allows achieving high performances straightforwardly. Indeed, despite the simple modeling of the system and the simple adopted optimization procedure, significant results have been obtained and the improvement with respect to the previously defined system-inversion-based methodology is evident. The readiness of the overall methodology makes it very suitable to be adopted in industrial environments, as demonstrated by the experimental results.

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