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G. Marro (I), A. Piazzi (I)

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A GEOMETRIC APPROACH TO ROBUST REGULATION

G. Marro and A. Piazzi

Department of Electronics, Computers and Systems, University of Bologna, Bologna, Italy

Abstract. Multivariable robust regulation is considered by using both the standard geometric approach tools and their extensions, i.e. the parameter-depending controlled invariants and the robust controlled invariants; in particular, the new concept of strong insensitivity to large parameter variations is introduced. This kind of approach is convenient since semilattice structures, as for regular controlled invariants, allow the computational effort to be reduced to just one basic algorithm (that for the supremum of the semilattice) which is a straightforward extension of the well-known basic recursive algorithm for the computation of the maximum controlled invariant contained in a given subspace in the non-robust case. Necessary conditions, stated in a very simple geometric form, are first established and a particular synthesis procedure is sketched.

Keywords. Robust regulation; controlled invariance; robust controlled invariance.

INTRODUCTION

Methods for the synthesis of robust regulators have been broadly investigated over the past 15 years, by using various techniques, ranging from qualitative-feedback theory to Wiener Hopf, H_2 and H_∞ sensitivity optimization.

As far as the state space approach is specifically considered, the problem of robust control has not stimulated a great deal of contributions: in this context the most significant results are certainly those by Davison *et alii*, who used matrix techniques without any resort to the definitions and tools of the geometric approach. This is quite surprising because, on the other hand, the multivariable regulator problem (without any robustness requirement) has been extensively investigated, starting from the early 70's, by Wonham (1973) and Francis (1977), till recent years, by Schumacher (1982) and the authors (1987b), and interesting results, stated in neat, intuitive geometric terms, have been obtained.

The aim of this paper is to propose a new methodology to include robustness in the geometric approach to the multivariable regulator problem. According to a definition generally adopted in the literature, we mean that a regulator is robust if it maintains the regulation property (i.e. that an error vector variable goes asymptotically to zero, whatever the initial conditions of the plant, regulator and exosystem are), in spite of parameter variations, provided generic stability and structure properties are preserved.

Use of the robust controlled invariant, introduced by the authors (1987), is the main difference between the approach herein developed and the previous ones. It is a very convenient tool, since it makes possible to derive very simple, general statements in strictly geometric terms, particularly when strong insensitivity to large parameter variations is considered.

STATEMENT OF THE PROBLEM

Consider the standard linear dynamic system

$$\dot{x}(t) = A(p)\,x(t) + B(p)\,u(t)\ , \qquad (1)$$
$$e(t) = E\,x(t)\ , \qquad (2)$$

where $x \in \mathcal{X} := \mathbb{R}^n$ denotes the state, $u \in \mathbb{R}^p$ the control input $e \in \mathbb{R}^q$ the controlled output and $p \in \Pi \subseteq \mathbb{R}^w$ a parameter set. Matrices $A(p)$, $B(p)$ and E are assumed to be partitioned as

$$A(p) = \begin{bmatrix} A_{11}(p) & A_{12}(p) \\ O & A_{22}(p) \end{bmatrix} ,$$
$$B(p) = \begin{bmatrix} B_1(p) \\ O \end{bmatrix} , \quad E = [\,E_1 \quad E_2\,]\ ; \qquad (3)$$

the state vector x is accordingly partitioned as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} , \qquad (4)$$

where x_1 denotes the state of the system to be controlled (*plant*), x_2 the state of the *exogenous subsystem*, a proper model for the exogenous signals: n_1 and n_2 will denote the corresponding dimensions. The pair $(A_{11}(p), B_1(p))$ is assumed to be controllable and $(A(p), E)$ to be observable for all admissible values of p. Note that the plant is a particular $A(p)$-invariant:

$$\mathcal{P} := \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \,:\, x_2 = 0 \right\} . \qquad (5)$$

Remark that matrix E is assumed to be constant. This assumption can easily be relaxed by means of a suitable change of basis if E depends on p, but $\mathcal{E} := \ker E$ is constant. In the new basis $\mathcal{P}$ may depend on the parameter.

Suppose system (1,2) is connected to a *regulator* modelled by:

$$\dot{z}(t) = N(p)\, z(t) + M(p)\, e(t) \ , \quad (6)$$

$$u = L(p)\, z(t) + K(p)\, e(t) \ , \quad (7)$$

where $z \in \mathbb{R}^m$ denotes the regulator state.

The *autonomous regulator problem with plant stability* is stated as follows: determine m and "nominal" matrices K, L, M, N such that the overall system for any value of p satisfies

i) the *regulation condition*:

$$\lim_{t\to\infty} e(t) = 0 \quad \forall\, x_1(0),\, x_2(0),\, z(0) \ ; \quad (8)$$

ii) the *plant stability condition*:

$$\lim_{t\to\infty} x_1(t) = 0 \, , \ \lim_{t\to\infty} z(t) = 0 \quad \forall\, x_1(0),\, z(0) \, , \quad \text{when } x_2(0) = 0 \, . \quad (9)$$

The regulator referred to is called "autonomous" because the exosystem is considered a part of the controlled system. Suppose for a moment that Π contains only one element, i.e. system and regulator are constant. Denote by A, B, K, L, M, N the corresponding "nominal" values of the matrices. Furthermore, suppose that all the exogenous modes are unstable. Let $\mathcal{J} := \max \mathcal{V}(A, \mathcal{B}, \mathcal{E})$ be the maximum $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{E}$ and $\mathcal{R}_{\mathcal{J}}$ the reachable subspace constrained on it. It is well known that $\mathcal{R}_{\mathcal{J}} = \mathcal{J} \cap \mathcal{K}$, where $\mathcal{K} := \min \mathcal{S}(A, \mathcal{E}, \mathcal{B})$ denotes the minimum $(A, \mathcal{E})$-conditioned invariant containing $\mathcal{B} := \mathrm{im} B$.

SOME BASIC RESULTS

First, we consider some results on the autonomous regulator at the nominal condition (with matrices not depending on p). A useful existence theorem, which provides a solvability characterization in terms of controlled invariants, is the following.

Theorem 1. The autonomous regulator problem admits a solution if and only if an $(A, \mathcal{B})$-controlled invariant $\mathcal{V}$ exists such that:

i) $\mathcal{V} \subseteq \mathcal{E}$;

ii) $\mathcal{V}$ is externally stabilizable;

iii) $\mathcal{V} \cap \mathcal{P}$ is internally stabilizable.

The proof is omitted here because the above theorem is a particular case of a more general result which has been proved by Basile, Marro, Piazzi (1987b). The constructive counterpart of Theorem 1 is stated as follows.

Theorem 2. Let all the exogenous modes be unstable: the autonomous regulator problem admits a solution if and only if

i) $\mathcal{J} + \mathcal{P} = \mathcal{X}$;

ii) $\mathcal{J} \cap \mathcal{P}$ is complementable with respect to $(\{0\}, \mathcal{J})$.

Condition *ii)* deserves an explanation: given three controlled invariants $\mathcal{V}_1$, $\mathcal{V}$ and $\mathcal{V}_2$, such that $\mathcal{V}_1 \subseteq \mathcal{V} \subseteq \mathcal{V}_2$, $\mathcal{V}$ is said to be *complementable with respect to the pair* $(\mathcal{V}_1, \mathcal{V}_2)$ if a controlled invariant $\mathcal{V}_c$ exists such that $\mathcal{V} + \mathcal{V}_c = \mathcal{V}_2$, $\mathcal{V} \cap \mathcal{V}_c = \mathcal{V}_1$. Complementability of a controlled invariant is related to the decomposition of a particular linear map and can easily be checked by means of the well-known Sylvester equation.

Proof of Theorem 2. (If) Because of *ii)* we infer the existence of an $(A, \mathcal{B})$-controlled invariant $\mathcal{V}_c$ such that:

$$\mathcal{J} \cap \mathcal{P} + \mathcal{V}_c = \mathcal{J} \ , \quad (10)$$

$$(\mathcal{J} \cap \mathcal{P}) \cap \mathcal{V}_c = \{0\} \ ; \quad (11)$$

hence

$$\mathcal{V}_c + \mathcal{P} = \mathcal{J} + \mathcal{P} \ , \quad (12)$$

$$\mathcal{V}_c \cap \mathcal{P} = \{0\} \ . \quad (13)$$

From (12) and *i)* it follows that $\mathcal{V}_c + \mathcal{P} = \mathcal{X}$. This equality is equivalent to $\mathcal{V}_c$ being externally stabilizable: in fact the well-known external stabilizability condition of a controlled invariant ($\mathcal{V}_c + \mathcal{R}$ to be an externally stable A-invariant, with $\mathcal{R}$ defined as the reachable set of the considered system) clearly holds, since in the present case, $\mathcal{R} = \mathcal{P}$. To complete the proof of necessity i.e. to fulfill all the conditions stated by Theorem 1, we note that $\mathcal{V}_c \subseteq \mathcal{E}$ by (10) and $\mathcal{V}_c \cap \mathcal{P}$ is internally stabilizable by (13).

(Only if) From condition *ii)* of Theorem 1 it follows that $\mathcal{J} + \mathcal{R} = \mathcal{J} + \mathcal{P}$ is externally stable, hence $\mathcal{J} + \mathcal{P} = \mathcal{X}$ since the exosystem is completely unstable. Let $\mathcal{V}$ be a resolvent satisfying all conditions of Theorem 1 and consider the change of basis in the state space defined by transformation $T := [T_1\, T_2\, T_3\, T_4\, T_5]$, with $\mathrm{im} T_1 = \mathcal{R}_{\mathcal{J}}$, $\mathrm{im}[T_1\, T_2] = \mathcal{V} \cap \mathcal{P}$, $\mathrm{im}[T_1\, T_2\, T_3] = \mathcal{J} \cap \mathcal{P}$, $\mathrm{im}[T_1\, T_2\, T_4] = \mathcal{V}$, $\mathrm{im}[T_1\, T_2\, T_3\, T_4] = \mathcal{J}$, $\mathrm{im}[T_1\, T_2\, T_3\, T_5] = \mathcal{P}$. Matrices $A' := T^{-1} A\, T$ and $B' := T^{-1} B$ have the following structures:

$$A' = \begin{bmatrix} A'_{11} & A'_{12} & A'_{13} & A'_{14} & A'_{15} \\ O & A'_{22} & A'_{23} & A'_{24} & A'_{25} \\ O & O & A'_{33} & O & A'_{35} \\ O & O & O & A'_{44} & O \\ A'_{51} & A'_{52} & A'_{53} & A'_{54} & A'_{55} \end{bmatrix} , \quad B' = \begin{bmatrix} B'_1 \\ O \\ O \\ O \\ B'_5 \end{bmatrix} .$$

The structure of B' is due to the inclusion $\mathcal{B} \subseteq \mathcal{P}$. Zero submatrices of A' are due to the particular structure of B' and to $\mathcal{R}_{\mathcal{J}}$ being an $(A, \mathcal{B})$-controlled invariant (in the second row), $\mathcal{V}$ being an $(A, \mathcal{B})$-controlled invariant (those in the third row) and $\mathcal{P}$ being an A-invariant (those in the fourth row). Consider the set of Sylvester equations

$$A'_{22}\, X_2 - X_2\, A'_{44} = -A'_{24} \, , \quad (14)$$

$$(A'_{11} + B'_1 F'_{11})\, X_1 - X_1\, A'_{44} = -A'_{14} - A'_{12}\, X_2 \, , \quad (15)$$

where F'_{11} is an arbitrary matrix chosen in such a way that $A'_{11} + B'_1 F'_{11}$ is a stable matrix (this is possible since pair (A'_{11}, B'_1) is controllable due to the reachability property of $\mathcal{R}_{\mathcal{J}}$). We also note that A'_{22} is stable because $\mathcal{V} \cap \mathcal{P}$ is internally stabilizable, and A'_{44} is completely ustable since its eigenvalues coincide with those of the exosystem. Therefore Sylvester equations (14,15) admit a unique solution (X_1, X_2). It follows that the subspace

$$\mathcal{V}_c := \mathrm{im}\left(\begin{bmatrix} X_1 \\ X_2 \\ O \\ I \\ O \end{bmatrix} \right) , \quad (16)$$

defined in the new basis, is an $(A, \mathcal{B})$-controlled invariant subspace. In fact, denote by $F'_{51}, F'_{52}, F'_{54}$ matrices such that

$$A'_{5i} + B'_5\, F'_{5i} = O \quad (i = 1, 2, 4) \ , \quad (17)$$

which exist, since $\mathcal{J}$ is a controlled invariant. Taking into account (14,15,17), by easy passages it follows that

$$A'\, \mathcal{V}_c = \mathrm{im}\left(\begin{bmatrix} X_1 A'_{44} - B'_1 F'_{11} X_1 \\ X_2 A'_{44} \\ O \\ A'_{44} \\ -B'_5 F'_{51} X_1 - B'_5 F'_{52} X_2 - B'_5 F'_{54} \end{bmatrix} \right) \subseteq \mathcal{V}_c + \mathcal{B}' \, .$$

By definition (16), $\mathcal{V}_c$ clearly satisfies $\mathcal{V}_c + \mathcal{P} = \mathcal{X}$ and $\mathcal{V}_c \cap \mathcal{P} = \{0\}$; performing the intersection of these relations with $\mathcal{J}$ yields, respectively, $(\mathcal{J} \cap \mathcal{P}) + \mathcal{V}_c = \mathcal{X}$ and $(\mathcal{J} \cap \mathcal{P}) \cap \mathcal{V}_c = \{0\}$, i.e. $\mathcal{J} \cap \mathcal{P}$ is complementable with respect to $(\{0\}, \mathcal{J})$. □

The following corollary, which is an immediate consequence of Theorem 2, appears as a refinement of Theorem 1.

Corollary 1. Let all the exogenous modes be unstable: the autonomous regulator problem admits a solution if and only if an $(A, \mathcal{B})$-controlled invariant $\mathcal{V}$ exists such that

i) $\mathcal{V} \subseteq \mathcal{E}$;

ii) $\mathcal{V} \oplus \mathcal{P} = \mathcal{X}$.

THE ROBUST CONTROLLED INVARIANT AND TWO NECESSARY CONDITIONS

Theorem 2 of the previous section will be used to derive some necessary conditions for the existence of a solution to the robust regulation problem.

In order to rigorously state results on robustness a preliminary characterization of the kind of robustness referred to is in order: a regulator is said to be *robust* with respect to the parameter set Π if the regulation condition (8) and the plant stability condition (9) hold for all $p \in \Pi$, while it is said to be *strongly insensitive to parameter variations* if the above conditions hold and, furthermore, for any initial state of the overall system and any value of $p \in \Pi$ which make $e(\cdot)$ zero, $e(\cdot)$ is preserved at zero also if p varies in time according to any piecewise continuous functions with values in Π.

We briefly recall the definition and main properties of the robust controlled invariant, which is the basic tool for handling strong insensitivity to parameter variations. Refer to differential equation (1).

Definition 1. Let $\mathcal{B}(p) := \mathrm{im} B(p)$. A subspace $\mathcal{V} \subseteq \mathbb{R}^n$ is called a *robust controlled invariant* relative to Π if

$$A(p)\, \mathcal{V} \subseteq \mathcal{V} + \mathcal{B}(p) \quad \forall p \in \Pi\ . \tag{18}$$

Property 1. Given a subspace $\mathcal{V} \subseteq \mathbb{R}^n$ and two real numbers t_0, t_1, $t_1 > t_0$, for any initial state $x(t_0) \in \mathcal{V}$ and any $p \in \Pi$ there exists at least one control function $u|_{[t_0,t_1]}$ such that the corresponding state trajectory $x|_{[t_0,t_1]}$ completely belongs to $\mathcal{V}$ if and only if $\mathcal{V}$ is a robust controlled invariant.

Since the sum of any two robust $(A(p), \mathcal{B}(p))$-controlled invariants is a robust $(A(p), \mathcal{B}(p))$-controlled invariant, the set of all robust controlled invariants contained in a given subspace $\mathcal{E}$ is a semilattice with respect to $+, \subseteq$, so that it admits a supremum. Denote it by $\max \mathcal{V}_r(A(p), \mathcal{B}(p), \mathcal{E})$ (maximum robust $(A(p), \mathcal{B}(p))$-controlled invariant contained in $\mathcal{E}$). The standard algorithm for computation of the well-known $\max \mathcal{V}(A, \mathcal{B}, \mathcal{E})$, the maximum $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{E}$, can be extended to compute the supremum of the new semilattice with robustness – see Basile and Marro (1987), Conte and Perdon (1989).

The following corollaries are straightforward consequences of Theorem 2 (they can easily be proved by contradiction).

Corollary 2. Let all the exogenous modes be unstable for all $p \in \Pi$. The robust regulation problem admits a solution only if

$$\mathcal{J}(p) + \mathcal{P}(p) = \mathcal{X} \quad \forall p \in \Pi\ . \tag{19}$$

Corollary 3. Let all the exogenous modes be unstable for all $p \in \Pi$. The regulation problem with strong insensitivity to parameter variations admits a solution only if

$$\mathcal{J}_r + \mathcal{P}(p) = \mathcal{X} \quad \forall p \in \Pi\ . \tag{20}$$

SOME RESULTS ON SYNTHESIS

Conditions (19) and (20) represent the main tools for the feasibility analysis of simply robust or strongly parameter insensitive regulation. In this section we will briefly present the lines along which a particular constructive solution to the simply robust regulation problem can be derived, provided (19) holds. An important further assumption we introduce for synthesis is that the plant is minimum-phase for all $p \in \Pi$. It is well-known that this is equivalent to assuming that all the unassignable eigenvalues of $\mathcal{J}(p)$ (i.e. those in between $\mathcal{R}_{\mathcal{J}(p)}$ and $\mathcal{J}(p)$) are stable.

Consider necessary condition (19): if it holds, by Corollary 1 there exists a controlled invariant $\mathcal{V}(p)$ such that

$$\mathcal{V}(p) \oplus \mathcal{P}(p) = \mathcal{X} \quad \forall p \in \Pi\ ; \tag{21}$$

the dimension of $\mathcal{V}(p)$ is clearly n_2. Let $V(p)$ be a basis matrix of $\mathcal{V}(p)$: because of (21) we can assume

$$V(p) = \begin{bmatrix} X_1(p) \\ I_{n_2} \end{bmatrix}, \tag{22}$$

where I_{n_2} denotes the $n_2 \times n_2$ identity matrix. Since $\mathcal{V}(p) \subseteq \mathcal{E}$, the equality

$$E_1\, X_1(p) + E_2 = O \tag{23}$$

holds for all admissible values of p. Furthermore, as $\mathcal{V}(p)$ is a controlled invariant, there exists a matrix $F(p)$ such that $(A(p) + B(p)F(p))\,\mathcal{V}(p) \subseteq \mathcal{V}(p)$. Taking into account matrix structures displayed in (3), this implies the existence of matrices $F_1(p), F_2(p)$ such that

$$\begin{aligned}(A_{11}(p) + B_1(p)F_1(p))\,X_1(p) + A_{12}(p)\\ + B_1(p)F_2(p) - X_1(p)A_{22}(p) = O\ .\end{aligned} \tag{24}$$

Note that in (24) we can assume $F_1(p) = O$ without any loss of generality since the choice of $F_1(p)$ is overcome by that of $F_2(p)$. Now we will perform the synthesis: the overall system is described by the extended equations

$$\dot{\hat{x}}(t) = \hat{A}(p)\, \hat{x}(t)\ , \quad e(t) = \hat{E}\, \hat{x}(t)\ , \tag{25}$$

where

$$\hat{A}(p) := \begin{bmatrix} A_{11}(p) + B_1(p)K(p)E_1 & A_{12}(p) + B_1(p)K(p)E_2 & B_1(p)L(p) \\ O & A_{22}(p) & O \\ M(p)E_1 & M(p)E_2 & N(p) \end{bmatrix},$$

$$\hat{E} := [\,E_1 \quad E_2 \quad O\,]\ .$$

As it is well-known to be necessary, we will adopt a regulator structure including an internal model of the exosystem – see Francis, Sebakhy and Wonham (1974, 1976); furthermore, we will synthesize a regulator having minimal order, i.e. having order n_2. Hence, we assume that A_{22} and N no longer depend on p and $N := A_{22}$. Let $\bar{p}$ denote the "nominal" value of the parameter (the one referred to for the synthesis) and $F_2(\bar{p})$ any matrix which satisfies (24) at $p = \bar{p}$ with $F_1(p) = O$. Assume $L(\bar{p}) := F_2(\bar{p})$. The other regulator nominal matrices $K(\bar{p})$ and $M(\bar{p})$ are chosen in such a way that the extended plant is stable. This is possible, provided that the plant has been assumed to be minimum-phase and completely controllable and observable, i.e. pole assignable by dynamic output-to-input feedback. For details, see Marro (1989).

It is easily checked that at $p = \bar{p}$ and assuming $X_2(\bar{p}) := I_{n_2}$

the subspace

$$\hat{\mathcal{W}} := \operatorname{im}\left(\begin{bmatrix} X_1(p) \\ I_{n_2} \\ X_2(p) \end{bmatrix} \right) , \tag{26}$$

is an externally stable $\hat{A}$-invariant contained in $\ker \hat{E}$. Its invariance is preserved also when p changes if the following set of equations admit a solution in $X_2(p)$ for all $p \in \Pi$:

$$B_1(p)\, F_2(p) = B_1(p)\, L(p)\, X_2(p) \ , \tag{27}$$

$$A_{22}\, X_2(p) = X_2(p)\, A_{22} \ . \tag{28}$$

These equations can be considered as ***robustness conditions***: the derived minimal-order regulator is robust if they admit a solution and the extended plant is stable for all $p \in \Pi$.

Still open questions are: *i)* how to derive a good estimate for the robustness domain Π (a domain where equations (27,28) and plant stability condition are satisfied) and *ii)* how to orient the choice of $\bar{p}$ and the free parameter of the regulator matrices in order to obtain a sufficiently large robustness domain or a maximal robustness domain.

CONCLUSIONS

When robust regulation is considered by state-space techniques, it is worth distinguishing simple robustness, which is handled by using parameter-depending controlled invariants, and strong insensitivity, which is approached by means of robust controlled invariants. In both cases the considered controlled system is connected to a controller whose matrices are constant or independently defined functions of the parameter. The efficiency of a particular regulation scheme is described by a *robustness domain*, where both a structural robustness condition, stated as the solvability of a given set of parameter-depending equations, and a standard stability condition, are satisfied.

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