Inferenza nella logica dei predicati del primo ordine

Inferenza nella logica del primo ordine
Sostituzioni e quantificatori

Sost(ϑ, α) applica la sostituzione ϑ alla formula α

Sost($\{x / Sam, y / Pam\}, Piace(x, y)\}$) = $Piace(Sam, Pam)$

Eliminazione degli universali

da $\forall x$ *Piace(x,Gelato)* sostituendo *{x/Ben}* si ha *Piace(Ben,Gelato)*

Eliminazione degli esistenziali

da $\exists x$ *Uccide(x,Vittima)* possiamo inferire *Uccide (Assa*sin *o,Vittima)*

(Assassino è una qualsisasi costante che non compare nella formula)

Introduzione degli esistenziali

da *Piace(Gerry,Gelato)* possiamo inferire $\exists x Piace(x, Gelato)$

Inferenza nella logica del primo ordine

Dalle formule :

$Buffalo(Bob)$

$Pig(Pat)$

$\forall x, y\ Buffalo(x) \wedge Pig(y) \Rightarrow Faster(x, y)$

si inferisce la nuova formula : $Buffalo(Bob) \wedge Pig(Pat)$

e col Modus Ponens : $Faster(Bob, Pat)$

MODUS PONENS

se c'è una sostituzione ϑ tale che Sost$(\vartheta, p_i) =$ Sost$(\vartheta, p_i') \forall i$

$$\frac{p_1', p_2', \ldots, p_n', (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{\text{Sost}(\vartheta, q)}$$

$p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q$ è una formula di Horn (un solo letterale positivo in una disgiunzione)

cioè : $\neg p_1 \vee \neg p_2 \neg \vee \ldots \neg \vee p_n \vee q$

Inferenza nella logica del primo ordine
UNIFICAZIONE

La funzione UNIFICA prende due formule atomiche *p* e *q* e restituisce una sostituzione che renda *p* e *q uguali.*

UNIFICA(p, q) = ϑ dove Sost(ϑ, p) = Sost(ϑ, q)

esempi :

UNIFICA *(Conosce(John,x),Conosce(John,Jane)) = {x/Jane}*

UNIFICA *(Conosce(John,x),Conosce(y,Leonid)) = {x/Leonid, y / John}*

UNIFICA *(Conosce(John,x),Conosce(y,Madre(y))) = {y/John, x / Madre(John)}*

UNIFICA *(Conosce(John,x),Conosce(x,Jane)) = fail*

Inferenza nella logica del primo ordine

Generalized Modus Ponens (GMP)

$$\frac{p_1', p_2', \ldots, p_n', (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\sigma} \quad \text{where } p_i'\sigma = p_i\sigma \text{ for all } i$$

E.g. p_1' = Faster(Bob,Pat)

p_2' = Faster(Pat,Steve)

$p_1 \wedge p_2 \Rightarrow q$ = $Faster(x, y) \wedge Faster(y, z) \Rightarrow Faster(x, z)$

σ = $\{x/Bob, y/Pat, z/Steve\}$

$q\sigma$ = $Faster(Bob, Steve)$

GMP used with KB of definite clauses (*exactly* one positive literal):
either a single atomic sentence or
(conjunction of atomic sentences) ⇒ (atomic sentence)
All variables assumed universally quantified

Inferenza nella logica del primo ordine

Forward chaining

When a new fact p is added to the KB
for each rule such that p unifies with a premise
if the other premises are known
then add the conclusion to the KB and continue chaining

Forward chaining is data-driven
e.g., inferring properties and categories from percepts

Forward chaining example

Add facts 1, 2, 3, 4, 5, 7 in turn.
Number in [] = unification literal; $\surd$ indicates rule firing

1. $Buffalo(x) \wedge Pig(y) \Rightarrow Faster(x,y)$
2. $Pig(y) \wedge Slug(z) \Rightarrow Faster(y,z)$
3. $Faster(x,y) \wedge Faster(y,z) \Rightarrow Faster(x,z)$
4. $Buffalo(Bob)$ [1a,×]
5. $Pig(Pat)$ [1b,$\surd$] → 6. $Faster(Bob,Pat)$ [3a,×], [3b,×] [2a,×]
7. $Slug(Steve)$ [2b,$\surd$]
   →8. $Faster(Pat,Steve)$ [3a,×], [3b,$\surd$]
      →9. $Faster(Bob,Steve)$ [3a,×], [3b,×]

Backward chaining

When a query q is asked
  if a matching fact q' is known, return the unifier
  for each rule whose consequent q' matches q
    attempt to prove each premise of the rule by backward chaining

(Some added complications in keeping track of the unifiers)

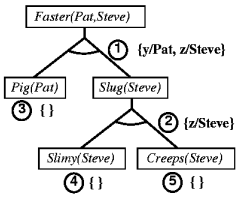
(More complications help to avoid infinite loops)

Two versions: find any solution, find all solutions

Backward chaining is the basis for logic programming, e.g., Prolog

Backward chaining example

1. $Pig(y) \wedge Slug(z) \Rightarrow Faster(y,z)$
2. $Slimy(z) \wedge Creeps(z) \Rightarrow Slug(z)$
3. $Pig(Pat)$ 4. $Slimy(Steve)$ 5. $Creeps(Steve)$



Completeness in FOL

Procedure i is complete if and only if

$KB \vdash_i \alpha$ whenever $KB \models \alpha$

Forward and backward chaining are complete for Horn KBs
but incomplete for general first-order logic

E.g., from

$PhD(x) \Rightarrow HighlyQualified(x)$
$\neg PhD(x) \Rightarrow EarlyEarnings(x)$
$HighlyQualified(x) \Rightarrow Rich(x)$
$EarlyEarnings(x) \Rightarrow Rich(x)$

should be able to infer $Rich(Me)$, but FC/BC won't do it

Does a complete algorithm exist?

A brief history of reasoning

450B.C.	Stoics	propositional logic, inference (maybe)
322B.C.	Aristotle	"syllogisms" (inference rules), quantifiers
1565	Cardano	probability theory (propositional logic + uncertainty)
1847	Boole	propositional logic (again)
1879	Frege	first-order logic
1922	Wittgenstein	proof by truth tables
1930	Gödel	∃ complete algorithm for FOL
1930	Herbrand	complete algorithm for FOL (reduce to propositional)
1931	Gödel	¬∃ complete algorithm for arithmetic
1960	Davis/Putnam	"practical" algorithm for propositional logic
1965	Robinson	"practical" algorithm for FOL—resolution

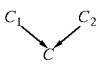
Resolution

Entailment in first-order logic is only semidecidable:
  can find a proof of α if $KB \models \alpha$
  cannot always prove that $KB \not\models \alpha$
Cf. Halting Problem: proof procedure may be about to terminate with success or failure, or may go on for ever

Resolution is a refutation procedure:
  to prove $KB \models \alpha$, show that $KB \wedge \neg\alpha$ is unsatisfiable

Resolution uses KB, $\neg\alpha$ in CNF (conjunction of clauses)

Resolution inference rule combines two clauses to make a new one:



Inference continues until an empty clause is derived (contradiction)

Resolution inference rule

Basic propositional version:

$$\frac{\alpha \vee \beta,\ \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or equivalently} \quad \frac{\neg\alpha \Rightarrow \beta,\ \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

Full first-order version:

$$\frac{p_1 \vee \ldots\ p_j\ \ldots \vee p_m,\quad q_1 \vee \ldots\ q_k\ \ldots \vee q_n}{(p_1 \vee \ldots\ p_{j-1} \vee p_{j+1}\ \ldots p_m \vee q_1 \ldots\ q_{k-1} \vee q_{k+1}\ \ldots \vee q_n)\sigma}$$

where $p_j\sigma = \neg q_k\sigma$

For example,

$$\frac{\neg Rich(x) \vee Unhappy(x),\quad Rich(Me)}{Unhappy(Me)}$$

with $\sigma = \{x/Me\}$

Conjunctive Normal Form

Literal = (possibly negated) atomic sentence, e.g., $\neg Rich(Me)$

Clause = disjunction of literals, e.g., $\neg Rich(Me) \vee Unhappy(Me)$

The KB is a conjunction of clauses

Any FOL KB can be converted to CNF as follows:
1. Replace $P \Rightarrow Q$ by $\neg P \vee Q$
2. Move $\neg$ inwards, e.g., $\neg\forall x\, P$ becomes $\exists x\, \neg P$
3. Standardize variables apart, e.g., $\forall x\, P \vee \exists x\, Q$ becomes $\forall x\, P \vee \exists y\, Q$
4. Move quantifiers left in order, e.g., $\forall x\, P \vee \exists x\, Q$ becomes $\forall x \exists y\, P \vee Q$
5. Eliminate $\exists$ by Skolemization (next slide)
6. Drop universal quantifiers
7. Distribute $\wedge$ over $\vee$, e.g., $(P \wedge Q) \vee R$ becomes $(P \vee Q) \wedge (P \vee R)$

Skolemization

$\exists x\ Rich(x)$ becomes $Rich(G1)$ where $G1$ is a new "Skolem constant"

$\exists k\ \frac{d}{dy}(k^y) = k^y$ becomes $\frac{d}{dy}(e^y) = e^y$

More tricky when $\exists$ is inside $\forall$

E.g., "Everyone has a heart"

$\forall x\ Person(x) \Rightarrow \exists y\ Heart(y) \wedge Has(x, y)$

Incorrect:

$\forall x\ Person(x) \Rightarrow Heart(H1) \wedge Has(x, H1)$

Correct:

$\forall x\ Person(x) \Rightarrow Heart(H(x)) \wedge Has(x, H(x))$

where H is a new symbol ("Skolem function")

Skolem function arguments: all enclosing universally quantified variables

Resolution proof

To prove α:
- negate it
- convert to CNF
- add to CNF KB
- infer contradiction

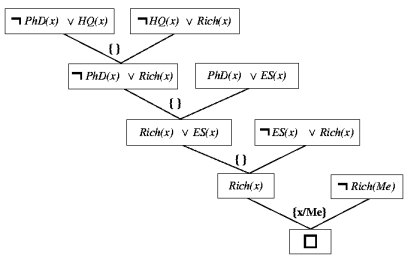
E.g., to prove $Rich(me)$, add $\neg Rich(me)$ to the CNF KB

$\neg PhD(x) \vee HighlyQualified(x)$
$PhD(x) \vee EarlyEarnings(x)$
$\neg HighlyQualified(x) \vee Rich(x)$
$\neg EarlyEarnings(x) \vee Rich(x)$

Refutazione (dimostrazione per assurdo)

$(KB \wedge \neg P \Rightarrow False) \Leftrightarrow (KB \Rightarrow P)$

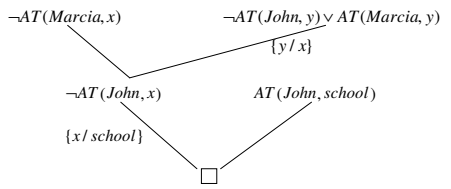
Resolution proof



Si consideri il problema *"se Marcia va ovunque vada John, e John è a scuola, dovè Marcia?"*

Assiomi :
$(\forall y) AT(John, y) \Rightarrow AT(Marcia, y)$
$AT(John, school)$
Domanda "*Dov' è Marcia ?*" $(\exists x) AT(Marcia, x)$
Negazione della domanda : $\neg(\exists x) AT(Marcia, x)$
Che si può scrivere come : $(\forall x) \neg AT(Marcia, x)$
In forma a clausole:
$\neg AT(John, y) \vee AT(Marcia, y)$
$AT(John, school)$

Inferenza nella logica del primo ordine



Sostituzione di risposta: $\{x / school\}$

Sostituendo nella domanda: $AT(Marcia, x)$