Minimum-time feedforward control of an open liquid container

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***Abstract*— The paper considers a minimum-time feedforward motion control problem for an open container carrying a liquid. The proposed solution is a time-continuous acceleration planning that avoids liquid spilling and satisfies amplitude constraints on jerk, acceleration, and velocity of the container moving on a linear guide of an automation line. This solution is based on linear programming and can provide rest-to-rest liquid motion planning or, alternatively, a rest-to-disequilibrium planning with bounded post-motion liquid oscillations. Experimental results on a test bench prototype show the effectiveness of the presented approach.**

I. Introduction

In the packaging industry, a typical problem is the transfer of an open container or package from the filling station to the sealing station of an automated packaging line [1]. When the container is partially filled with a liquid, the automated transfer may be critical due to the liquid slosh induced by the container motion. Indeed, a faster transfer may improve the line productivity but it could cause a splash out of liquid that is unacceptable. Hence, it arises the practical need of achieving a fast container transfer while controlling and keeping limited the resulting liquid slosh.

An in-depth investigation of liquid sloshing in moving containers was carried out by NASA research [2], [3] with the aim of studying the fuel sloshing in the motion control of rockets. For various container geometries, this research provided the basic equations governing the motion of the free liquid surface inside the container and the deduction of natural frequencies of the oscillatory liquid modes. Many subsequent researches on liquid sloshing dynamics have been reported in [4], [5] also covering nonlinear and multimodal modeling.

This sloshing modeling has been the starting point of many control methods addressing the transfer of an open liquid container. A class of methods uses feedback control with some sensing of the liquid surface displacement. Relevant works include: sliding mode control [6], [7]; H_∞ control with optimal command input [8], [9]; control based on a hybrid shape approach [10]; generalized PI control based on differential flatness [11].

However, as Grundelius and Bernhardsson [12] pointed out, feedforward control of the open liquid container may be a better approach for the packaging industry. This corrisponds to design a kinematic motion planning for the open container and then to implement it without the need of a feedback sensor measuring the displacement of the free liquid surface. A feedforward control based on an infinite impulse response filter is presented in [13]. Approaches using the input shaping command idea are reported in [14], [15]. A minimum-time acceleration profile for the container motion is presented in [1] with constraints on maximum acceleration and elevation of the liquid surface. In this work, the structure of the time-optimal acceleration profile is deduced from process experience and the profile parameters are determined with an a search numerical procedure. A common assumption of these feedforward approaches is to model the slosh dynamics as a linear second-order system.

In this paper, still using a simple second-order slosh model, we propose a minimum-time feedforward control of the container by exploiting the idea of generalized bang-bang control proposed in [16]. The method is to plan a minimum-time continuous acceleration profile with a bounded jerk (i.e., the acceleration derivative) and amplitude constraints on the liquid elevation as well as on the velocity and acceleration of the container. This approach can allow a rest-to-rest motion for the liquid inside the container or a rest-to-disequilibrium motion with an amplitude constraint on the post-motion liquid oscillations. The latter case may be acceptable when the residual liquid oscillations do not interfere with the closing of the container in the automation line and it can be preferred because it further reduces the optimal transfer time.

The paper is organized as follows. Section II briefly describes the slosh dynamics inside a moving cilindrical container. The formulation of the minimum-time feedforward control problem and its solution based on linear programming are reported in Section III. Experimental results using a test bench prototype are presented in Section IV. Section V ends the paper with concluding remarks.

Notation (piecewise-continuity): A function $f : \mathbb{R} \to \mathbb{R}$, $t \to f(t)$ has PC^0 continuity, and we say $f \in PC^0$ if $f \in C^0(\mathbb{R} - \{t_1, t_2, \dots\})$ and there exist $\lim_{t \to t_i^-} f(t)$, $\lim_{t \to t_i^+} f(t)$, $i = 1, 2, \dots$. Here $\{t_1, t_2, \dots\}$ is a set of discontinuos time-instants. Function f has PC^1 continuity and we say $f \in PC^1$ if $f \in C^0(\mathbb{R})$ and its derivative function Df has PC^0 continuity.

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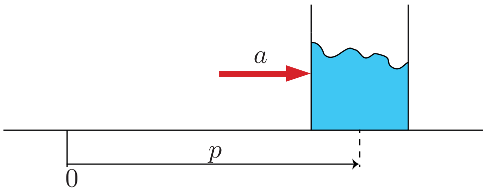


Fig. 1. Liquid sloshing inside a moving cylindric container.

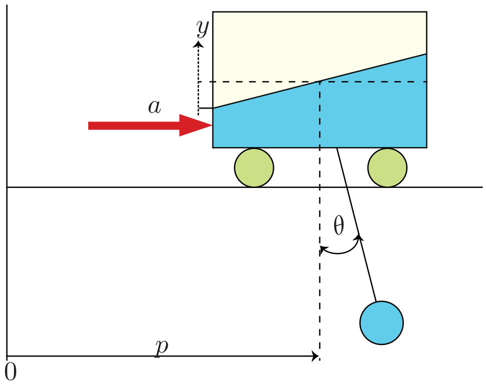


Fig. 2. Cart-pendulum system representing the dynamics of the first asymmetric sloshing mode.

II. Slosh Modeling

In this section, we briefly discuss the slosh dynamics inside an open container carring a liquid substance. Figure 1 illustrates a cylindric liquid container, whose center is at position p on a longitudinal axis, subject to an external acceleration, a. The cylinder contains a liquid, which oscillates during the motion of the container. A mathematically rigorous description of the liquid sloshing dynamics is based on the Navier-Stokes equations, a set of nonlinear, partial differential equations. The resulting motion of the liquid is described by the superposition of various sloshing modes, characterized by different oscillation frequencies. For sloshing control purposes (see for instance [9] or [17]), it is generally sufficient to take into account only the first asymmetric mode (i.e. the asymmetric mode with the lowest frequency). With this approximation, the slosh dynamics is equivalent to that of a cart-pendulum (see figure 2).

The generalized coordinates of the cart-pendulum system are the position p of the cart center of mass, which corresponds to the position of the container, and the pendulum angle θ, which represents the angle between the horizontal line and the liquid surface (assumed to be a plane). The system control input is given by the cart acceleration $a = \ddot{p}$ and the output y is defined as the vertical displacement of the free liquid surface with respect to the rest level at the left side of the container. This output is related to the pendulum angle by expression $y = R\tan\theta$. The equations of motion can be obtained by considering the equation of the pendulum angular momentum:

$$\begin{cases} \ddot{\theta}(t) = -\frac{g}{l}\sin\theta(t) + \frac{a(t)}{l}\cos\theta(t) - c\dot{\theta}(t) \\ y = R\tan\theta(t) \end{cases} \tag{1}$$

where l is the pendulum length, c is a viscous friction coefficient and g is the gravity acceleration.

Assuming that the angle θ is small, we can linearize (1) on the equilibrium $\theta = \dot{\theta} = 0$, obtaining the linear model

$$\begin{cases} \ddot{\theta}(t) = -\frac{g}{l}\theta(t) + \frac{a(t)}{l} - c\dot{\theta}(t) \\ y(t) = R\theta(t)\,. \end{cases} \tag{2}$$

In system (2), the transfer function $T(s)$ between the acceleration input u and the liquid vertical displacement y is given by

$$T(s) = \frac{R}{g}\frac{1}{1+\frac{lc}{g}s+\frac{l}{g}s^2}. \tag{3}$$

It is convenient to rewrite $T(s)$ as

$$T(s) = K\frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2} \tag{4}$$

where $K = \frac{R}{g}$ is the static gain, $\omega_n = \sqrt{\frac{g}{l}}$ is the natural frequency and $\delta = \frac{c}{2}\sqrt{\frac{l}{g}}$ is the damping ratio. Since K is known, $T(s)$ is fully determined by parameters ω_n and δ.

According to [4] the natural frequency of oscillation ω_n of the first asymmetric mode of a liquid contained in a cylinder is given by

$$\omega_n = \sqrt{\frac{g\cdot\xi}{R}\tanh\left(\frac{\xi\cdot h}{R}\right)} \tag{5}$$

where R is the tank radius, h is the liquid depth, g is the gravity acceleration and $\xi \simeq 1.841$ is a constant specific to this mode. In [2], the damping coefficient is approximated by:

$$\delta = 0.79\sqrt{Re}\left[1+\frac{0.318}{\sinh\left(\frac{1.84h}{R}\right)}\left(1+\frac{1-\frac{h}{R}}{\cosh\left(\frac{1.84h}{R}\right)}\right)\right] \tag{6}$$

where Re is the reverse Reynolds number

$$Re = \frac{\nu}{\sqrt{g\cdot(2\cdot R)^3}} \tag{7}$$

and ν is the kinematic viscosity of the liquid.

III. MINIMUM-TIME CONSTRAINED FEEDFORWARD CONTROL

In this section, we formulate the minimum-time constrained feedforward control problem and present an approximate solution based on time-discretization and linear programming.

A. Problem formulation

Consider a state-space realization of transfer function (4):

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}a(t) \\ y(t) = \mathbf{c}\mathbf{x}(t)\,, \end{cases} \tag{8}$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\delta\omega_n \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$\mathbf{c} = \begin{bmatrix} K\omega_n^2 & 0 \end{bmatrix}.$$

The minimum-time constrained feedforward control problem addressed in this paper can be introduced in two versions as follows (Problem 1 and Problem 2 below).

Let $p_f > 0$ be the final desired position of the container, v_M, a_M, j_M be the assigned maximum values of the container velocity, acceleration and jerk, and y_M be the assigned maximum value of the liquid surface vertical displacement from the equilibrium (liquid) surface. The container velocity and position can be defined according to $v(t) := \int_0^t a(\xi)d\xi$ and $p(t) := \int_0^t v(\xi)d\xi$.

Problem 1: Find the minimum transition time t_f and the time-optimal acceleration function $a(t) \in PC^1$ such that model equations (8) and conditions below are satisfied.

- Liquid equilibrium at the initial time:

$$\mathbf{x}(0) = \mathbf{0}. \tag{9}$$

- Kinematic constraints on the container motion:

$$a(0) = 0, \tag{10}$$

$$0 \leq v(t) \leq v_M,\ \forall t \in [0, t_f], \tag{11}$$

$$|a(t)| \leq a_M,\ \forall t \in [0, t_f], \tag{12}$$

$$|\dot{a}(t)| \leq j_M,\ \forall t \in [0, t_f], \tag{13}$$

$$p(t_f) = p_f,\ v(t_f) = 0,\ a(t_f) = 0. \tag{14}$$

- Amplitude constraint on the liquid oscillations:

$$|y(t)| \leq y_M, \forall t \in [0, t_f]. \tag{15}$$

- Liquid equilibrium at the final time:

$$\mathbf{x}(t_f) = \mathbf{0}\,. \tag{16}$$

Problem 2: Find the minimum transition time t_f and the time-optimal acceleration function $a(t) \in PC^1$ such that model equations (8) and conditions below are satisfied.

- Conditions (9)-(14) as in Problem 1.
- Amplitude constraint on the liquid oscillations during and after the container motion:

$$|y(t)| \leq y_M, \forall t \in [0, t_f + t_s] \tag{17}$$

where t_s is a settling time that can be fixed to $\frac{2\pi}{\omega_n}$.
Solutions of problems 1 and 2, i.e, the optimal transfer time and corrisponding time-optimal acceleration, are denoted by t_f^* and $a^*(t)$.

Note that in Problem 1 and 2 it necessary to impose both conditions $y(t) \leq y_M$ and $y(t) \geq -y_M$ to prevent liquid overspilling at the left and right container walls. This is correct due the assumption that the free liquid surface is a plane during the liquid oscillations.

Remark 1: Removing the constraint of final liquid equilibrium as it has been done in Problem 2 allows a faster container transfer, at the expenses of post-final output oscillations that, anyway, do not cause liquid spilling due to the extended constraint (17). This can be useful to improve productivity in applications in which the sealing process to close the container is undisturbed by the residual liquid oscillations.

B. Reduction to linear programming

Problems 1 and 2 can be conveniently reformulated by introducing the following augmented system whose state vector $\hat{\mathbf{x}}$ is defined as $(\mathbf{x}, p, v, a)$ and control input $u(t)$ is $\dot{a}(t)$, i.e., the jerk of the container motion:

$$\begin{cases} \dot{\hat{\mathbf{x}}}(t) = \hat{\mathbf{A}}\hat{\mathbf{x}}(t) + \hat{\mathbf{b}}u(t) \\ y(t) = \hat{\mathbf{c}}\hat{\mathbf{x}}(t)\,, \end{cases} \tag{18}$$

with

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{b} \\ \mathbf{0} & 0 & 1 & 0 \\ \mathbf{0} & 0 & 0 & 1 \\ \mathbf{0} & 0 & 0 & 0 \end{bmatrix}, \hat{\mathbf{b}} = \begin{bmatrix} \mathbf{0} \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$\hat{\mathbf{c}} = \begin{bmatrix} \mathbf{c} & 0 & 0 & 0 \end{bmatrix}.$$

Problems 1 and 2 are then equivalent to optimizations

$$\min_{u(t) \in PC^0} t_f \tag{19}$$

such that

$$|u(t)| \leq j_M,\ 0 \leq v(t) \leq v_M,\ |a(t)| \leq a_M,\ \forall t \in [0, t_f], \tag{20}$$

$$\hat{\mathbf{x}}(0) = \mathbf{0}, \tag{21}$$

and (for problem 1)

$$|y(t)| \leq y_M,\ \forall t \in [0, t_f], \tag{22}$$

$$\hat{\mathbf{x}}(t_f) = [\mathbf{0}\, p_f\, 0\, 0]^T, \tag{23}$$

or (for problem 2)

$$|y(t)| \leq y_M,\ \forall t \in [0, t_f + t_s], \tag{24}$$

$$[\mathbf{0}\,\mathbf{I}_3]\hat{\mathbf{x}}(t_f) = [p_f\, 0\, 0]^T. \tag{25}$$

We now assume that the input signal u is obtained from a discrete-time signal $\tilde{u}(k)$, $k \in \mathbb{Z}$ applied to a first-order hold filter, i.e.

$$u(t) = \tilde{u}(\lfloor \frac{t}{T} \rfloor)\,,$$

where $T > 0$ is the sampling time and $\lfloor x \rfloor = \max_{k \in \mathbb{Z}}\{k \leq x\}$ is the integer part of the real number x.

The sampled state $\mathbf{z}(k) = \hat{\mathbf{x}}(kT)$ and the sampled output $\tilde{y}(k) = y(kT)$, for $k \in \mathbb{Z}$, are the solution of the discretized system

$$\begin{cases} \mathbf{z}(k+1) = \mathbf{A}_d\mathbf{z}(k) + \mathbf{b}_d\tilde{u}(k) \\ \tilde{y}(k) = \mathbf{c}_d\mathbf{z}(k)\,, \end{cases} \tag{26}$$

where $\mathbf{A}_d = e^{\hat{\mathbf{A}}T}$, $\mathbf{b}_d = \int_0^T e^{\hat{\mathbf{A}}\xi}\hat{\mathbf{b}}d\xi$, $\mathbf{c}_d = \hat{\mathbf{c}}$.

For a given integer number k, we denote by $N^k \in \mathbb{R}^{5 \cdot k \times k}$ the Toeplitz matrix

$$\mathbf{N}^k = \begin{bmatrix} \mathbf{b}_d & 0 & \dots & 0 \\ \mathbf{A}_d\mathbf{b}_d & \mathbf{b}_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \mathbf{A}_d^{k-1}\mathbf{b}_d & \dots & \mathbf{A}_d\mathbf{b}_d & \mathbf{b}_d \end{bmatrix}.$$

Then, introduce

$$\mathbf{M}_y^k := diag(\mathbf{c}_d, \ldots, \mathbf{c}_d)\mathbf{N}^k, \quad (27)$$
$$\mathbf{M}_v^k := diag(\mathbf{c}_v, \ldots, \mathbf{c}_v)\mathbf{N}^k, \quad (28)$$
$$\mathbf{M}_a^k := diag(\mathbf{c}_a, \ldots, \mathbf{c}_a)\mathbf{N}^k, \quad (29)$$

where $\mathbf{c}_v = [0\,0\,0\,1\,0]$ and $\mathbf{c}_a = [0\,0\,0\,0\,1]$.

With this notation in mind, we define the following discrete-time counterparts of problems 1 and 2.

Find the smallest number of discrete-time steps k_f for which there exists a vector $\mathbf{u} := [\tilde{u}(0) \ldots \tilde{u}(k_f-1)] \in \mathbb{R}^{k_f}$ such that the following linear programming problem admits a solution (corresponding to an approximate solution of problem 1)

$$\begin{cases} -j_M \mathbf{1}_{k_f} \leq \mathbf{u} \leq j_M \mathbf{1}_{k_f} \\ \mathbf{0}_{k_f} \leq \mathbf{M}_v^{k_f} \mathbf{u} \leq v_M \mathbf{1}_{k_f} \\ -a_M \mathbf{1}_{k_f} \leq \mathbf{M}_a^{k_f} \mathbf{u} \leq a_M \mathbf{1}_{k_f} \\ -y_M \mathbf{1}_{k_f} \leq \mathbf{M}_y^{k_f} \mathbf{u} \leq y_M \mathbf{1}_{k_f} \\ \left[\mathbf{A}_d^{k_f-1}\mathbf{b}_d, \mathbf{A}_d^{k_f-2}\mathbf{b}_d, \ldots, \mathbf{b}_d\right] \mathbf{u} = [\mathbf{0}\, p_f\, 0\, 0]^T \end{cases} \quad (30)$$

or (corrisponding to an approximate solution of problem 2)

$$\begin{cases} -j_M \mathbf{1}_{k_f} \leq \mathbf{u} \leq j_M \mathbf{1}_{k_f} \\ \mathbf{0}_{k_f} \leq \mathbf{M}_v^{k_f} \mathbf{u} \leq v_M \mathbf{1}_{k_f} \\ -a_M \mathbf{1}_{k_f} \leq \mathbf{M}_a^{k_f} \mathbf{u} \leq a_M \mathbf{1}_{k_f} \\ -y_M \mathbf{1}_{k_f+k_s} \leq \mathbf{M}_y^{k_f+k_s} [\mathbf{u}\, \mathbf{0}_{k_s}]^T \leq y_M \mathbf{1}_{k_f+k_s} \\ [\mathbf{0}\, \mathbf{I}_3] \left[\mathbf{A}_d^{k_f-1}\mathbf{b}_d, \mathbf{A}_d^{k_f-2}\mathbf{b}_d, \ldots, \mathbf{b}_d\right] \mathbf{u} = [p_f\, 0\, 0]^T \end{cases} \quad (31)$$

where $k_s := \lfloor \frac{2\pi}{\omega_n T} \rfloor$. Here, notation $x \leq y$ for vectors $x, y \in \mathbb{R}^n$ denotes the element-wise inequality (i.e. $x_i \leq y_1$, $i = 1, \ldots, n$) and $\mathbf{0}_k$, $\mathbf{1}_k$ denote the k-dimensional vectors whose components are all equal to 0 and 1 respectively.

The optimal solution k_f^* and corresponding $\mathbf{u}^*$ for problems (30) and (31) can be found by solving a sequence of linear programming tests. This sequence can be simply generated by a bisection algorithm (cf. [16] for details).

IV. Experiments

A. Experimental setup

Figure 3 is a schematic representation of the key elements of the test bench prototype. The cylindrical container is inserted into a rigid frame, which is connected to a pulley actuated by a brushless electric motor. The liquid vertical displacement at the container wall is measured by a laser range sensor. Figure 4 shows the actual prototype, that was provided by Zanelli S.r.l., a company based in Parma, specialized in the production of machines and plants for the production and the packaging of coatings , sealants, padding and pastes. This prototype allows a container transfer till to the total distance of 0.38 m. The brushless motor is an Omron Accurax G5, with a rated torque of 2.39 Nm, a rated speed of 3000 turns/min and a rated power output of 750 W. The motor is equipped with a 20-bit incremental encoder. The motion is controlled by an Omron Sysmac 501-1500 PLC, configured with a sampling time of 1 ms.

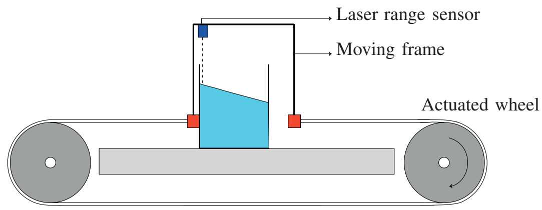


Fig. 3. Basic scheme of the prototype.

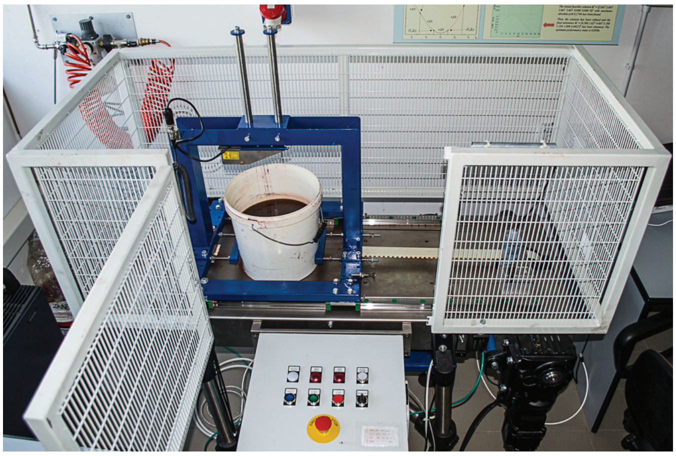


Fig. 4. Picture of the test bench prototype.

The cylindric container has a radius $R = 0.108$ m. The height of the liquid at rest is $h = 0.149$ m. We have considered liquids of different dynamic viscosities: water (1 cP), lubricant oil (3000 cP), paint (5000 cP). The laser range sensor is a Sick OD-250W150I, with a response time of 1 ms and an accuracy of 1.2 mm. This sensor allows a very fast and precise measurement of the height of liquid at the container wall. However, it operates correctly only on liquids with a sufficiently high reflection coefficient. For this reason, iron oxide has been added to water and oil to increase its reflectivity. This has not been necessary for paint. Note that this sensor is not used for feedback control, but only for system identification and performance evaluation.

B. Direct parameter identification

For each liquid, we have performed 5 identification experiments with different container acceleration profiles. The corrisponding liquid vertical displacement, that is the output y of system (4), has been recorded to obtain the natural frequency ω_n and the damping ratio δ by standard system identification methods.

The following table shows a comparison of the values for ω_n and δ given by the theoretical formulas (5), (6) with the values obtained by the identification procedures, for the three different used types of liquid.

Liquid	Theoretical		Identification	
	$\omega_n\left(\frac{rad}{s}\right)$	δ	$\omega_n\left(\frac{rad}{s}\right)$	δ
water	12.8888	0.0014	12.5687	0.01252
oil	12.8888	0.0866	12.6072	0.04211
paint	12.8888	0.1000	13.5030	0.1913

In the case of water and oil, the values of the natural frequency obtained with identification methods are very close to the theoretical values given by (5). Paint exhibits a larger difference, perhaps due to the fact that this liquid, with a very high viscosity and a non-Newtonian behavior, presents sloshing dynamics influenced by significant nonlinear effects, not taken into account by (5). The identified values of the damping ratio exhibit large differences from the theoretical ones prescribed by (6). In the case of oil or paint, this is probably related to the intrinsic difficulty of measuring a single value of viscosity for non-Newtonian fluids. In the case of water, the difference can be due to the presence of iron oxide in the water, added to increase its refraction coefficient to facilitate the correct measurements of the laser range sensor.

C. Experimental Results

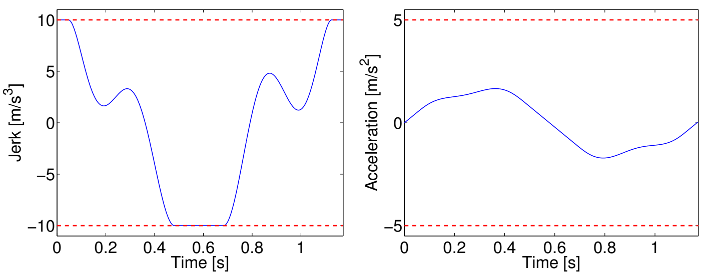
In the planning of the time-optimum feedforward control we have considered a container transfer given by distance $p_f = 0.35\,m$. Kinemetics constraints on velocity and acceleration are given by $v_M = 0.62\,\frac{m}{s}$ and $a_M = 5\,\frac{m}{s^2}$. To prevent liquid from spilling, the constraint on liquid elevation is $y_M = 0.035\,m$. The time-discretization is set with sampling time $T = 0.004\,s$ and the model parameters are those obtained by system identification. The time-optimal jerk profiles have been then determinated by standard linear programming routines.

1) Experiment 1 (Water): We have set the the input jerk bound as $j_M = 10\frac{m}{s^3}$. The obtained minimum-times are $t_f^* = 1.176\,s$ and $t_f^* = 1.144\,s$ for Problem 1 (rest-to-rest planning) and 2 (rest-to-disequilibrium planning) respectively. For the rest-to-rest planning Figure 5, Figure 6, and Figure 7 show the time-optimal jerk and acceleration profiles, the corresponding command velocity profile, and the simulated and measured liquid dispacement respectively.

From these figures, it is possible to see that the prescribed bounds and the final rest condition are essentially satisfied. It is significant to note that, despite the modeling simplifications made, there is a strong similarity between the measured and the expected simulated output. The spikes appearing in figure 7 are due to occasionally incorrect measurements of the laser range sensor. These are caused by the low reflectivity of water, despite the addition of iron oxide.

2) Experiment 2 (Oil): In this case, the input jerk bound is chosen as $j_M = 30\frac{m}{s^3}$. The obtained minimum-times are $t_f^* = 1.024\,s$ and $t_f^* = 0.920\,s$ for Problem 1 and 2 respectively. For the rest-to-disequilibrium planning, Figure 8, Figure 9, and Figure 10 show the time-optimal jerk and acceleration profiles, the corresponding command velocity profile, and the simulated and measured liquid dispacement respectively.

3) Experiment 3 (Paint): The input jerk bound is again chosen as $j_M = 30\frac{m}{s^3}$. The obtained minimum-times are $t_f^* =$



(a) Jerk profile. (b) Acceleration profile.

Fig. 5. Experiment 1: time-optimal planned motion.

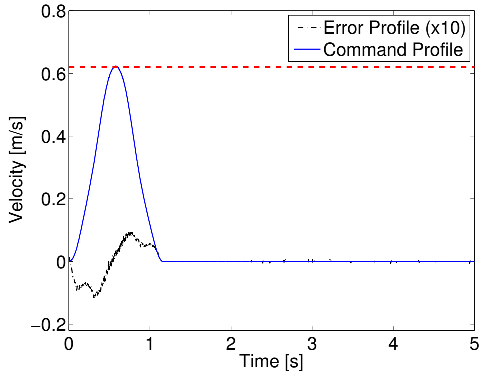


Fig. 6. Experiment 1: time-optimal velocity profile command and implementation error.

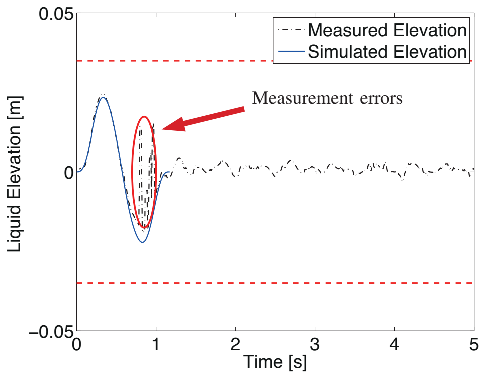
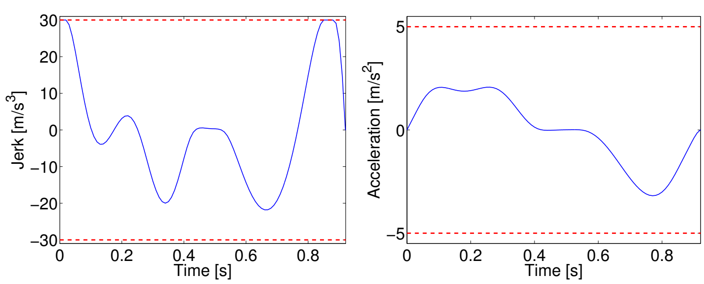


Fig. 7. Experiment 1: measured and simulated liquid elevation (output signal $y(t)$).



(a) Jerk profile. (b) Acceleration profile.

Fig. 8. Experiment 2: time-optimal planned motion.

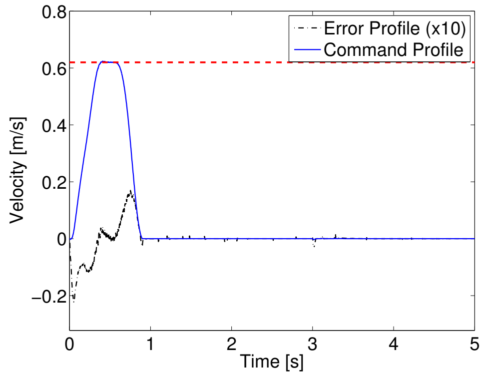


Fig. 9. Experiment 2: time-optimal velocity profile command and implementation error.

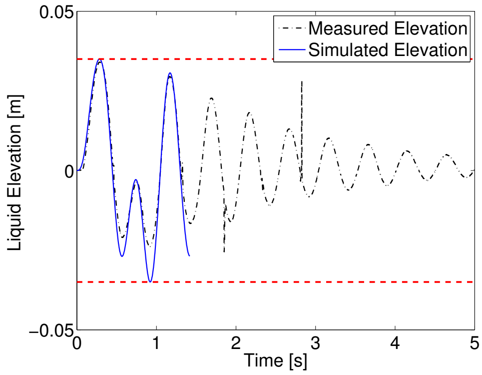


Fig. 10. Experiment 2: measured and simulated liquid elevation (output signal ($y(t)$).

$1.004\,s$ and $t_f^* = 0.892\,s$ for Problem 1 and 2 respectively. To save space the time-optimal plottings are not displayed.

V. Conclusions

In this paper we have proposed a new approach for the minumum-time feedforward control of an open liquid container. All the relevant possible constraints are considered also comprising a selectable bound on the maximal allowed container jerk. The presented solution is based on linear programming and can provide rest-to-rest liquid motion planning or, alternatively, a rest-to-disequilibrium planning with bounded post-motion liquid oscillations. This latter case is interesting to further reduce the transfer times of open liquid containers and may be adopted in automation lines of the packaging industry.

VI. Acknowledgments

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