Various Issue on Mapping and Vehicle Reference

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This document try to explain several issue on vehicle dynamics and how to control it on large environment, like earth.

1 Mapping on Earth

First of all in order to work with waypoint it is important to describe how to convert waypoint in GPS coordinate to local coordinate and vice-versa.

The model of earth is a Geoid, formally really similar to an ellipsoid. GPS coordinate system uses a standard, called WGS 84, to describe earth, and any value of latitude and longitude provided by a GPS have to be related to this coordinate system.

It is possible to descrive a point on earth using a classical cartesian notation X,Y,Z: the Earth-centred Earth-fixed (ECEF) or conventional terrestrial coordinate system rotates with the Earth and has its origin at the centre of the Earth. In this reference frame, the X axis passes through the equator at the prime meridian, the Z axis passes through the north pole and the Y axis can be determined by the right-hand rule to be passing through the equator at 90 deg longitude.

It is possible to use the following equation to converte a point from Geodetic to ECEF (Earth Centered, Earth Fixed Cartesian) reference frame:

$$\begin{aligned} X &= (N+h)\cos\phi\cos\lambda \\ Y &= (N+h)\cos\phi\sin\lambda \\ Z &= [N(1-e^2)+h]\sin\phi \end{aligned} \tag{1}$$

where ϕ, λ, h are geodetic latitude, longitude and heigh above ellipsoid, X, Y, Z tridimensional earth centered cartesian coordinates. Since earth is not spherical the radius of curvature in prime vertical N is not a costant and it values

$$N(\phi) = \frac{a}{\sqrt{1-e^2\sin^2\phi}} \tag{2}$$

where a is the semi-major earth axis (ellipsoid equatorial radius), b the semi-minor earth axis (ellipsoid polar radius). This value are calculated using flattening $f = \frac{a-b}{b}$ and squared eccentricy $e^2 = 2f - f^2$.

Constant	Value	Unit	Meaning
a	6378137.0	m	WGS-84 Earth semimajor axis
b	6356752.3142	m	WGS-84 Earth semiminor axis
f	3.3528107×10^{-3}		ellipsoid flatness
$1/f$	298.257223563		reciprocal of flattening
e	8.1819191×10^{-2}		eccentricity
e^2	$6.69437999013 \times 10^{-3}$		first numerical eccentricy

Consider that there is no closed form solution for transformation from ECEF to LLA (Latitude, Longitude, Altitude) if the altitude is not zero.

In many targeting and tracking applications the local East, North, Up (ENU) Cartesian coordinate system is far more intuitive and practical than ECEF or Geodetic coordinates. The local ENU coordinates are formed from a plane tangent to the Earth's surface fixed to a specific location and hence it is sometimes known as a "Local Tangent" or "local geodetic" plane. To keep this system easy to use we align the (x) axis with true East and the (y) axis to true North. The resulting East-North-Up system is a right handed rectangular coordinate system as shown in the figure 2.

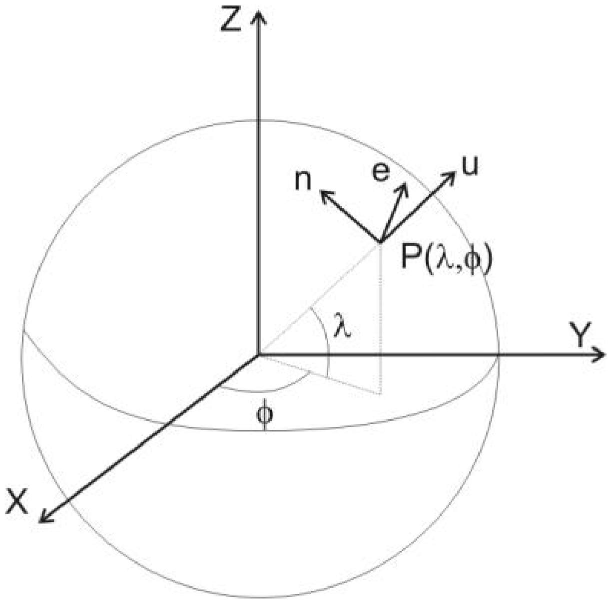

Figure 1: LLA, ECEF and ENU coordinate systems

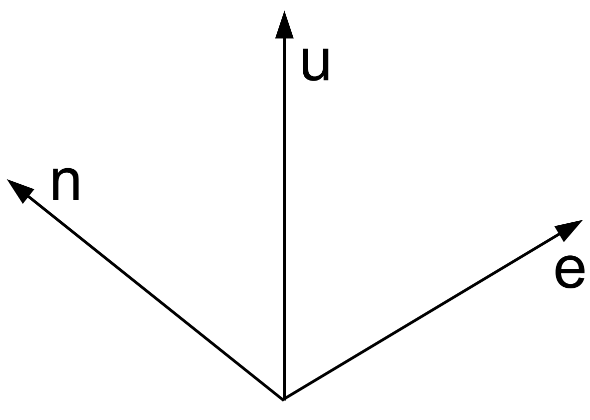

Figure 2: East-North-Up (ENU) system

It is desired to further transform the ECEF coordinates into what will be termed here the Local Tangent Plane (LTP). To pass from ECEF to local topocentric:

$$\begin{bmatrix} n \\ e \\ u \end{bmatrix} = \begin{bmatrix} -\sin\lambda\cos\phi & -\sin\lambda\sin\phi & \cos\lambda \\ -\sin\phi & \cos\phi & 0 \\ \cos\lambda\cos\phi & \cos\lambda\sin\phi & \sin\lambda \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{3}$$

Let $\mathbf{x}_0$ be the origin of the LTP expressed in ECEF coordinates. Transforming the origin from the center of the Earth to the Tangent Plane origin is accomplished by subtracting $\mathbf{x}_0$ from the ECEF coordinate. Call the ECEF coordinate $\mathbf{x}$, and the new translated coordinate system $\mathbf{x}'$.

To convert small changes in latitude, longitude and height into ECEF coordinates we need to Taylor expand equation 1. To simplify equation it is possible to ignore $\partial N(\phi)$ and subsitute the result in 3.